



Research Paper

# Approaches To Analysing Micro-Drivers Of Aggregate Productivity

Australia

2019

1351.0.55.164

AUSTRALIAN BUREAU OF STATISTICS

ABS Catalogue No. 1351.0.55.164

© Commonwealth of Australia 2019

This work is copyright. Apart from any use as permitted under the *Copyright Act 1968*, no part may be reproduced by any process without prior written permission from the Commonwealth. Requests and inquiries concerning reproduction and rights in this publication should be addressed to The Manager, Intermediary Management, Australian Bureau of Statistics, Locked Bag 10, Belconnen ACT 2616, by telephone (02) 6252 6998, fax (02) 6252 7102, or email: <intermediary.management@abs.gov.au>.

In all cases the ABS must be acknowledged as the source when reproducing or quoting any part of an ABS publication or other product.

Produced by the Australian Bureau of Statistics.

## INQUIRIES

For further information about these and related statistics, contact the National Information and Referral Service on 1300 135 070.



New  
Issue

**Research Paper**

**Approaches To  
Analysing Micro-Drivers Of  
Aggregate Productivity**

**Methodology Transformation Branch**

Methodology Division

AUSTRALIAN BUREAU OF STATISTICS

EMBARGO: 11.30 AM (CANBERRA TIME) MON 25 MAR 2019

# APPROACHES TO ANALYSING MICRO-DRIVERS OF AGGREGATE PRODUCTIVITY

Chien-Hung Chien<sup>1,2</sup>, A.H. Welsh<sup>2</sup>, and Robert Breunig<sup>3</sup>

<sup>1</sup>ABS Methodology Transformation Branch

<sup>2</sup>ANU Mathematical Science Institute

<sup>3</sup>ANU Crawford School of Public Policy

## 1 EXECUTIVE SUMMARY

This study describes the patterns of productivity growth across eighteen industries. We examine the components of this productivity growth by estimating the contribution of entry, exit, within-firm growth and re-allocation to productivity growth in Australia in the period 2002 - 2013.

We use an experimental linked dataset of 10 million workers across 1.5 million firms. We produce industry-level estimates using firm-level data across 18 industries. We estimate worker- and firm-specific effects using a grouping algorithm appropriate for sparse matrices.

We find that firm entry and exit are by far the largest contributors to productivity growth across all industries. In general, firm exit contributes positively to productivity growth whereas firm entry generally contributes negatively. This would suggest that policies which facilitate firm entry and exit are likely to help achieve increased productivity gains. Policies which provide large advantages to incumbent firms are likely to detract from productivity growth.

# APPROACHES TO ANALYSING MICRO-DRIVERS OF AGGREGATE PRODUCTIVITY

Chien-Hung Chien<sup>1,2</sup>, A.H. Welsh<sup>2</sup>, and Robert Breunig<sup>3</sup>

<sup>1</sup>ABS Methodology Transformation Branch

<sup>2</sup>ANU Mathematical Science Institute

<sup>3</sup>ANU Crawford School of Public Policy

## 2 ABSTRACT

We estimate the contribution of entry, exit, within-firm growth and re-allocation to productivity growth in Australia in the period 2002 - 2013. We use an experimental linked dataset of 10 million workers across 1.5 million firms. We produce industry-level estimates using firm-level data across 18 industries. We estimate worker- and firm-specific effects using a grouping algorithm appropriate for sparse matrices. Firm entry and exit are by far the largest contributors to productivity growth across all industries. In general, firm exit contributes positively to productivity growth whereas firm entry generally contributes negatively.

---

**Disclaimer:** the results of these studies are based, in part, on tax data supplied by the Australian Taxation Office (ATO) to the ABS under the Taxation Administration Act 1953, which requires that such data is only used for the purpose of administering the Census and Statistics Act 1905. Legislative requirements to ensure privacy and secrecy of this data have been adhered to. In accordance with the Census and Statistics Act 1905, results have been confidentialised to ensure that they are not likely to enable identification of a particular person or organisation. This study uses a strict access control protocol and only a current ABS officer has access to the underlying microdata.

Any findings from this paper are not official statistics and the opinions and conclusions expressed in this paper are those of the authors. The ABS takes no responsibility for any omissions or errors in the information contained here. Views expressed in this paper are those of the authors and do not necessarily represent those of the ABS. Where quoted or used, they should be attributed clearly to the authors.

.....

CONTENTS

<b>1 EXECUTIVE SUMMARY</b>	<b>1</b>
<b>2 ABSTRACT</b>	<b>2</b>
<b>3 INTRODUCTION</b>	<b>4</b>
<b>4 LITERATURE REVIEW</b>	<b>4</b>
<b>5 DATA DESCRIPTION</b>	<b>5</b>
5.1 Data confidentiality . . . . .	5
5.2 Data processing . . . . .	5
<b>6 STATISTICAL MODELS</b>	<b>6</b>
6.1 Worker equation . . . . .	6
6.2 Firm level productivity model . . . . .	7
6.3 Industry productivity . . . . .	8
<b>7 ESTIMATION METHODS</b>	<b>10</b>
7.1 Data structure . . . . .	10
7.2 Preconditioned conjugate gradient algorithm . . . . .	11
7.3 Identification using grouping algorithm . . . . .	12
7.3.1 Issues in Identification . . . . .	13
<b>8 EMPIRICAL RESULTS</b>	<b>17</b>
8.1 Firm dynamics and aggregate productivity . . . . .	17
8.2 Firm level model results . . . . .	18
8.3 Worker level model results . . . . .	18
<b>9 CONCLUSIONS AND FUTURE DIRECTIONS</b>	<b>20</b>
<b>10 REFERENCES</b>	<b>21</b>
<b>APPENDIX A INDUSTRY DECOMPOSITION</b>	<b>24</b>
<b>APPENDIX B FIRM MODEL RESULTS</b>	<b>27</b>
<b>APPENDIX C SUMMARY STATISTICS</b>	<b>32</b>
<b>11 ACKNOWLEDGEMENTS</b>	<b>34</b>

.....

### 3 INTRODUCTION

Lentz and Mortensen (2010) describe firms as dynamic: firms are born, some grow old, and all eventually die. Their mortality, like children, is high while they are young. Those who survive initially grow rapidly, but only a few continue to flourish. New entrants tend to be smaller and less productive on average, but they are more diverse than the surviving firms. The diversity will diminish as firms mature and eventually the market will be responsible for their death. An efficient market allocates resources from less productive firms to more productive ones but there are different mechanisms that can influence aggregate productivity.

The seminal surveys by Bartelsman and Doms (2000) and Syverson (2011) discuss the advantages of using microdata to better understand the determinants of aggregate productivity. Aggregate statistics, which give a good overview of trends in productivity growth, do not show the variability that occurs at micro levels. It is important to develop a good understanding of the degree to which different aspects of productivity growth within and across firms contribute to different productivity growth across industries.

This study describes the patterns of productivity growth across eighteen industries. We examine the components of this productivity growth by looking at firm entry and exit, reallocation across continuing firms and productivity growth within firms. We also examine whether these patterns differ across industries?

Our industry level results are decomposed into contributions from surviving, entering and exiting firms. We apply linear models, estimated separately by industry, using a Cobb Douglas production function as the basis to estimate firm level productivity. Previous studies have shown the importance of correcting for endogeneity in estimating productivity due to strong correlation between inputs and outputs in the production process. We adapt the approaches of Mare et al. (2016) and Abowd et al. (2002) to estimate labour inputs which we use to address endogeneity.

This paper is structured as follows: section three provides the literature review, section four describes the data, section five presents the statistical models, section six contains empirical results and the final section gives some conclusions and future directions for further research.

### 4 LITERATURE REVIEW

Developing a good understanding of the determinants of aggregate productivity is challenging because the economy is complex. One factor in aggregate productivity growth is the reallocation of resources from more productive firms to less productive ones. Part of this effect is captured by firm entry and exit. Several studies describe the role of the reallocation of resources between firms. Influential work by Olley and Pakes (1996) and Bartelsman and Dhrymes (1998) developed the principal methods that most economists use to measure the impact of firm dynamics on aggregate productivity. These methods are often used in analyses to better understand the process of creative destruction that can occur within and between sectors of the economy (Foster et al., 2001).

Lafrance and Baldwin (2011) explored the contribution firm turnover has on productivity growth in the Canadian services industries. They found that the market naturally allocates resources from uncompetitive firms to new entrants. Nguyen and Hansell (2014) explored the firm dynamic effects on productivity growth for Australian manufacturing and business services industries. They have found that entering and exiting firms make smaller contributions to overall productivity than established firms.

Economists also consider productivity differences to come from better measures of inputs used in the production process. Labour economists have observed strong correlations between the differences in firm



.....

productivity and wage costs per worker (Lentz and Mortensen, 2010). However, this strong correlation can potentially cause endogeneity (Fox and Smeets, 2011). Better labour quality measures for production are important to minimise endogeneity in productivity analysis (Foster et al., 2001).

This study explores the effects of firm dynamics on aggregate productivity by adapting approach of Mare et al. (2016). The labour component is estimated using the approach of Abowd et al. (2002) which takes into account two-sided worker and firm effects. This estimated labour component is then used in a firm production function equation. The contributions to the aggregate industry productivity are derived using the approaches of Griliches and Regev (1995) and Melitz and Polanec (2015) to take into account firm dynamics.

## 5 DATA DESCRIPTION

The Australian Taxation Office (ATO), Australia Business Register (ABR) and ABS datasets are held in both the Business Longitudinal Analytical Data Environment (BLADE) for firms (ABS and DIIS, 2017) and the prototype Graphically Linked Information Discovery Environment for workers (Chien and Mayer, 2015). This section describes the ABS confidentiality protocol and the data processing carried out for this study. The sample period is between 2002-03 to 2012-13.

### 5.1 Data confidentiality

The ATO data is provided to the Australian Statistician under the Taxation Administration Act 1953 and (ABR) data is supplied to the Australian Statistician under A New Tax System (Australian Business Number) Act 1999. These Acts require that these data are only used by the ABS for administering the Census and Statistics Act 1905. The ABS is obliged to maintain the confidentiality of individuals and businesses in these ATO and ABR datasets, as well as comply with provisions that govern the use and release of this information, including the Privacy Act 1988 ABS (2015).

This study uses a strict access control protocol. Access to the datasets includes audit trails and is limited on a need to know basis. All ABS officers are legally bound to secrecy under the Census and Statistics Act 1905. Officers sign an undertaking of fidelity and secrecy to ensure that they are aware of their responsibilities. The ABS policies and guidelines govern the disclosure of information to maintain the confidentiality of individuals and organisations. This study presents only aggregate results to ensure that they are not likely to enable identification of a worker or a firm.

### 5.2 Data processing

The study uses a similar linking strategy to ABS (2015). The firm records were deterministically linked using Australian Business Numbers (ABNs) and worker records were deterministically linked using De-identified and Encoded Tax File Numbers. As the linking variable is encrypted, it is not possible to identify individuals in the datasets.

The worker characteristics such as age, sex and occupation come from Personal Income Tax (PIT) and the wage information comes from Pay-As-You-Go (PAYG). PAYG contains a longer time series than PIT so this study backcasts the PIT data to the same length. In particular, the earliest available PIT information is used to backcast sex (holding it constant) and age (by subtracting 1 year). Two methods to backcast the skill categories for workers were explored, either using the average or holding it constant for each worker. This study found that it is not appropriate to use the average worker skill categories because workers tend to become more skilled over time so using average skill inflates the worker's skill level. The ABS Australian and New Zealand Standard Classification of Occupations was used to convert

.....

occupations into a 5-scale skills categorical variable for the analysis (ABS, 2009). We stress that the prototype worker panel data is constructed for research purposes only.

BLADE contains firm characteristics data. It does not contain any data about worker characteristics beyond the number of employees and total wages. We include all firms. If we restrict our study to firms with more than 2 employees the results are mostly as reported. Summary statistics can be found in Appendix Appendix C.

We use an unbalanced panel of firms. Dropping firms where some variables are missing results in a dramatic reduction in sample size. Therefore we assume missing at random and impute missing variables for non-missing firms using a sequential regression approach in SAS, namely the `proc mi` procedure. We create 10 imputed data sets upon which we base our estimation. We then reproduce this analysis 10 times and we select the results which maximise the likelihood function for the firm-level productivity model (3) below from the 10 imputations.

Results from our imputation approach match ABS results more closely than those where we drop all firms with any missing values. The analysis of the complete case data, which involves dropping 80 per cent of the data, produces a lot of volatility and inconsistency with ABS results therefore we prefer the imputation approach.

## 6 STATISTICAL MODELS

### 6.1 Worker equation

This analysis uses a modified wage equation adapted from Abowd et al. (2002). The worker panel is unbalanced, meaning that the available observations for each worker  $i$ ,  $i = 1, \dots, N$  can be different. Suppose that the observations for worker  $i$  are available at time  $t = 1, \dots, T_i$ . So  $t = 1$  is the first time period and  $t = T_i$  is the last time period for the available observations for worker  $i$ . Note that there can be gaps—a worker might appear in periods 1 and 3 but not in period 2, for example. We model  $y_{it}$ , the wages for worker  $i$  at time  $t$ , as

$$\ln(y_{it}) = \mathbf{x}_{it}^\top \boldsymbol{\alpha} + \theta_i + \mathbf{f}_{it}^\top \boldsymbol{\psi} + \epsilon_{it}, \quad (1)$$

where  $\mathbf{x}_{it}$  is a  $p$ -vector of characteristics of worker  $i$  at time  $t$ ,  $\boldsymbol{\alpha}$  is a  $p$ -vector of unknown coefficients of the worker characteristics,  $\theta_i$  represents unobserved (time-invariant) worker effects, the components of the  $J$ -vector  $\boldsymbol{\psi} = (\psi_1, \dots, \psi_J)^\top$  represent firm effects (e.g. specific factors such as pay structure that affect workers' wages),  $\mathbf{f}_{it}^\top = (f_{i1t}, \dots, f_{iJt})^\top$  is a firm indicator vector with components

$$f_{ijt} = \begin{cases} 1, & \text{if worker } i \text{ works for firm } j \text{ at time } t \\ 0, & \text{otherwise,} \end{cases}$$

and the random disturbances  $\epsilon_{it}$  are assumed to satisfy  $\epsilon_{it} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma^2)$ .

It is convenient to write the term  $\mathbf{x}_{it}^\top$  which describes worker characteristics in Wilkinson and Rogers (1973) notation as  $Sex + HighSkill + MediumSkill + WorkingSkill + Time + Poly(Age, 4) + Sex : Poly(Age, 4) + Sex : Time$ . Here the indicator  $Sex = 1$  if worker  $i$  is male and 0 otherwise. The indicator  $HighSkill = 1$  if worker  $i$  has a tertiary qualification and 0 otherwise. The indicator  $MediumSkill = 1$  if worker  $i$  has at most a diploma qualification and 0 otherwise. The indicator  $WorkingSkill = 1$  if worker

.....

$i$  has at most a certificate III qualification and 0 otherwise. Workers with qualifications lower than a certificate III qualification are treated as the baseline and included in the intercept. The variable *Time* is represented by 11 time indicator variables, one for each year with 2001 – 02 as baseline. The variable *Age*, the age of worker  $i$  at time  $t_{it}$ , is fitted by a quartic polynomial including linear, quadratic, cubic and quartic functions. We include a quartic function to better describe the data because fitting only quadratic and cubic terms does not describe the decline in workers' wage as they get older. We include the interaction terms  $Sex : Poly(Age, 4)$  between *Sex* and *Age* and  $Sex : Time$  between *Sex* and *Time*. This makes each  $\mathbf{x}_{it}^\top \boldsymbol{\alpha}$  a sum of  $p = 34$  terms.

Following Mare et al. (2016), we estimate (1) pooling across all workers at all time periods in all industries. We then derive an instrument for firm-specific labour inputs, which we use in (3) below, based upon the average fitted values for each firm  $j$

Specifically, let  $\widehat{\boldsymbol{\alpha}}, \widehat{\theta}_1, \dots, \widehat{\theta}_N$  and  $\widehat{\boldsymbol{\psi}}$  denote estimates of parameters in (1). Then the proposed instrumental variable is

$$\widehat{z}_t^{(j)} = \overline{\mathbf{x}}_t^{-(j)\top} \widehat{\boldsymbol{\alpha}} + \overline{\theta}_t^{-(j)} + \widehat{\boldsymbol{\psi}}_j, \quad (2)$$

where

$$\overline{\mathbf{x}}_t^{-(j)} = \frac{\sum_{i=1}^N f_{ijt} \mathbf{x}_{it}}{\sum_{i=1}^N f_{ijt}} \quad \text{and} \quad \overline{\theta}_t^{-(j)} = \frac{\sum_{i=1}^N f_{ijt} \widehat{\theta}_i}{\sum_{i=1}^N f_{ijt}}.$$

Note that the variables in (2) now have a firm superscript  $j$  to reflect the averaging of worker effects within each firm  $j$ . When we want to emphasise below that firm  $j$  belongs to industry  $k$ , we also include the industry superscript  $k$  so that the estimated firm average worker effect  $\widehat{z}_t^{(j)}$  in (2) becomes  $\widehat{z}_t^{(jk)}$ .

## 6.2 Firm level productivity model

The firm volume outputs can be modelled as functions of the observed inputs such as capital, materials and labour in volume terms, and unobserved components in the production process (Fox and Smeets, 2011). We use a Cobb Douglas production function, similar to Breunig and Wong (2008) and Mare et al. (2016) to model  $y_{jkt}^*$ , the value added (i.e. sales adjusted for repurchase of stock) deflated by industry Gross Value Added implicit price deflators by firm  $j$  in industry  $k$  at time  $t$  ABS (2018a), as

$$\ln(y_{jkt}^*) = \beta_k + \beta_{1k} \ln L_{jkt} + \beta_{2k} \ln K_{jkt} + \beta_{3k} \ln M_{jkt} + \beta_{4k} \ln Age_{jkt}^* + \tau_{kt} + \varepsilon_{jkt}, \quad (3)$$

where  $\ln L_{jkt}$  is the logarithm of labour inputs,  $\ln K_{jkt}$  is the logarithm of cost of capital that includes depreciation, capital rental expenses and capital work deductions deflated by the industry consumption of fixed capital implicit price deflators ABS (2018a). The logarithm of material costs  $\ln M_{jkt}$  that are the inputs used in the production deflated by Producer Price Indexes Intermediate Goods ABS (2018b). The logarithm of firm age is  $\ln Age_{jkt}^*$ . We also include different intercepts  $\beta_k$  for each industry and time fixed effects  $\tau_{kt}$ . The multi-factor productivity terms  $\varepsilon_{jkt}$  are assumed to satisfy  $\varepsilon_{jkt} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_k^2)$  to estimate unbiased coefficients for the Cobb Douglas production function Zellner et al. (1966).

The endogeneity issue when estimating production functions has been well documented in the literature.

.....

This problem is caused by firms increasing inputs used for production in response to temporary increases in demand for their products and services (Eberhardt and Helmers, 2010). The temporary positive correlation between input and output makes it difficult to distinguish if an increase in firm production is caused by increased firm productivity or a temporary firm response to increased demand. As a result, the temporary positive correlation causes bias in estimating the production function equation. Many studies use predicted values from instrumental variables equations, i.e. using lagged inputs as instruments for the current inputs, to mitigate the bias (Gandhi et al., 2011). For example, Olley and Pakes (1996) and Breunig and Wong (2008) used lagged capital investment; Levinsohn and Petrin (2003) and Bakhtiari (2015) used lagged material inputs; Fox and Smeets (2011) used lagged wage costs as instrumental variables.

However, Reed (2015) cautioned against the use of lagged instrumental variables to correct for the simultaneity bias. Instead of using lagged inputs as instruments, we followed a similar approach to Mare et al. (2016) by aggregating the worker level to firm level for labour inputs to correct for this bias. Our labour component comes from the averaged worker effects  $\hat{z}_t^{(jk)}$  from (2). (As we are fitting separate models for each industry, we include  $k$  to emphasise the nesting of firm  $j$  in industry  $k$ .) The model fitted to the data is

$$\ln(y_{jkt}^*) = \beta_k + \beta_{1k}\hat{z}_t^{(jk)} + \beta_{2k}\ln K_{jkt} + \beta_{3k}\ln M_{jkt} + \beta_{4k}\ln Age_{jkt}^* + \tau_{kt} + \varepsilon_{jkt}. \quad (4)$$

### 6.3 Industry productivity

Mare et al. (2016) defined industry weights firm productivity  $\hat{P}_{jkt}$  and  $\hat{w}_{jkt}$  from (4) as

$$\hat{P}_{jkt} = \hat{\beta}_k + \hat{\tau}_{kt} + \hat{\varepsilon}_{jkt} \quad \text{and} \quad \hat{w}_{jkt} = \hat{\beta}_{1k}\hat{z}_t^{(jk)} + \hat{\beta}_{2k}\ln K_{jkt} + \hat{\beta}_{3k}\ln M_{jkt} + \hat{\beta}_{4k}\ln Age_{jkt}^*.$$

The estimated parameters for (4) include the industry intercepts  $\hat{\beta}_k$ ; the labour inputs  $\hat{\beta}_{1k}$ ; the cost of capital  $\hat{\beta}_{2k}$ ; materials costs  $\hat{\beta}_{3k}$ ; firm age  $\hat{\beta}_{4k}$ ; time fixed effects  $\hat{\tau}_{kt}$  and multi-factor productivity

$$\hat{\varepsilon}_{jkt} = \ln(y_{jkt}^*) - (\hat{\beta}_k + \hat{\beta}_{1k}\hat{z}_t^{(jk)} + \hat{\beta}_{2k}\ln K_{jkt} + \hat{\beta}_{3k}\ln M_{jkt} + \hat{\beta}_{4k}\ln Age_{jkt}^* + \hat{\tau}_{kt}).$$

They then defined the aggregate productivity index  $A_{kt}$  for an industry  $k$  at time  $t$  as

$$A_{kt} = \hat{w}_{jkt}^* \hat{P}_{jkt}, \quad \text{where} \quad \hat{w}_{jkt}^* = \frac{\hat{w}_{jkt}}{\sum_{j=1}^{J_{kt}} \hat{w}_{jkt}} \quad (5)$$

and  $J_{kt}$  is the number of firms in industry  $k$  at time  $t$ . Note that the weights  $\hat{w}_{jkt}^*$  satisfy  $\sum_{j=1}^{J_{kt}} \hat{w}_{jkt}^* = 1$  for each industry  $k$  and time  $t$ .

Next, aggregating to industry level, let  $\hat{w}_{kt} = \sum_{j=1}^{J_{kt}} \hat{w}_{jkt}$  and  $\hat{P}_{kt} = \sum_{j=1}^{J_{kt}} \hat{P}_{jkt}$ . Then the aggregate productivity index  $A_t$  for all industries at time  $t$  is

$$A_t = \widehat{w}_{kt}^{**} \widehat{P}_{kt}, \quad \text{where} \quad \widehat{w}_{kt}^{**} = \frac{\widehat{w}_{kt}}{\sum_{k=1}^{K_t} \widehat{w}_{kt}}, \quad (6)$$

and  $K_t$  is the number of industries at time  $t$ . Note the weights  $\widehat{w}_{kt}^{**}$  satisfy  $\sum_{k=1}^{K_t} \widehat{w}_{kt}^{**} = 1$  for each time  $t$ .

Griliches and Regev (1995) proposed decomposing the changes in aggregate productivity from time  $t-1$  to  $t$  into contributions from surviving (S), entering (EN) and exiting (EX) firms as

$$\Delta A_{kt} = W_{kt} + B_{kt} + EN_{kt} + EX_{kt}, \quad (7)$$

$$\begin{aligned} \text{where} \quad W_{kt} &= \sum_{j \in S_{kt}} \bar{w}_{jk} \Delta \widehat{P}_{jkt}, \quad B_{kt} = \sum_{j \in S_{kt}} \Delta \widehat{w}_{jkt} (\bar{P}_{jk} - \bar{A}_k), \quad EN_{kt} = \sum_{j \in EN_{kt}} \widehat{w}_{jkt} (\widehat{P}_{jk} - \bar{A}_k), \text{ and} \\ EX_{kt} &= \sum_{j \in EX_{kt}} \widehat{w}_{jkt-1} (\widehat{P}_{jkt-1} - \bar{A}_k). \end{aligned}$$

The symbol  $\Delta$  represents changes so  $\Delta A_{kt} = A_{kt} - A_{kt-1}$  is the change in aggregate productivity for industry  $k$  from time  $t-1$  to time  $t$ . Bars represent averages between  $t$  and  $t-1$  so  $\bar{w}_{jk} = \frac{(\widehat{w}_{jkt} + \widehat{w}_{jkt-1})}{2}$  and  $\bar{A}_k = \frac{(A_{kt} + A_{kt-1})}{2}$ . The definition of surviving  $j \in S_{kt}$ , entering  $j \in EN_{kt}$  and exiting firms  $j \in EX_{kt}$  is based on the firm transitions on an annual basis over the observed sample period. Survivors are firms operating in  $t$  and  $t-1$ , exiting firms are firms that exist at time  $t-1$  but not at time  $t$  and entering firms are firms that did not exist at time  $t-1$  but did at time  $t$ . The contribution of the surviving firms is decomposed into two components, the within industry reallocation  $W_{kt}$  that measures the change in firm productivity weighted by the average of the weights at  $t$  and  $t-1$ , i.e.  $\bar{w}_{jk}$ , and the between industry reallocation  $B_{kt}$  that measures deviations from the average productivity, i.e.  $\bar{A}_k$ , including the impact of firm entry and exit (Foster et al., 2001).

Fox and Smeets (2011) discussed the importance of using appropriate benchmarks to calculate the contribution of surviving, entering and exiting firms to aggregate productivity. This study explores an alternative method proposed by Melitz and Polanec (2015). Their dynamic Olley-Pakes decomposition incorporates a decomposition proposed by Olley and Pakes (1996), which captures the covariance of productivity changes and market share of an individual firm over time. Let  $J$  denotes  $J_{kt}$  i.e. the number of firms in industry  $k$  at time  $t$  and the equation is

$$\begin{aligned} A_{kt}^* &= \bar{P}_{kt} + \frac{1}{(J-1)} \sum_{j=1}^J (\widehat{w}_{jkt} - \bar{w}_{kt}) (\widehat{P}_{jkt} - \bar{P}_{kt}) \\ &= \bar{P}_{kt} + Cov(\widehat{w}_{jkt}, \widehat{P}_{jkt}), \end{aligned} \quad (8)$$

$$\text{where} \quad \bar{P}_{kt} = \frac{\sum_j I_{jkt} \widehat{P}_{jkt}}{\sum_{j=1}^N I_{jkt}}, \quad \text{and} \quad \bar{w}_{kt} = \frac{\sum_j I_{jkt} \widehat{w}_{jkt}}{\sum_{j=1}^N I_{jkt}} \quad \text{with} \quad I_{jkt} = \begin{cases} 1, & \text{if firm } j \text{ operates in industry } k \text{ at time } t \\ 0, & \text{otherwise.} \end{cases}$$

.....

The dynamic Olley-Pakes approach decomposes aggregate productivity into contributions from surviving, entering and exiting firms as

$$\Delta A_{kt}^* = W_{kt}^* + B_{kt}^* + EN_{kt}^* + EX_{kt}^*, \quad (9)$$

where  $W_{kt}^* = \Delta \bar{P}_{kt}$ ,  $B_{kt}^* = \Delta Cov_{kt}$ ,  $EN_{kt}^* = \sum_{j \in EN} \hat{w}_{jkt} (A_{jkt \in EN} - A_{jkt \in S})$ , and

$$EX_{kt}^* = \sum_{j \in EX} \hat{w}_{jkt} (A_{jkt \in EX} - A_{jkt-1 \in S}).$$

The dynamic Olley-Pakes decomposition approach uses more appropriate benchmarks for the entering and exiting firms (see discussion in the empirical results section). For example, entering firms only generate positive growth when they have higher productivity than surviving firms at time  $t$ . Similarly, exiting firms can only generate a positive contribution if they have lower productivity than surviving firms at time  $t-1$ .

## 7 ESTIMATION METHODS

### 7.1 Data structure

The model in (1) can be written as a model for each worker  $i$  by stacking the observations over time. We obtain

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\alpha} + \mathbf{1}_i \theta_i + \mathbf{F}_i \boldsymbol{\psi} + \boldsymbol{\epsilon}_i, \quad (10)$$

where

$$\mathbf{y}_i = \begin{bmatrix} y_{it_{i1}} \\ \vdots \\ y_{it_{iT_i}} \end{bmatrix}, \quad \mathbf{X}_i = \begin{bmatrix} \mathbf{x}_{it_{i1}}^\top \\ \vdots \\ \mathbf{x}_{it_{iT_i}}^\top \end{bmatrix}, \quad \mathbf{1}_i = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \mathbf{F}_i = \begin{bmatrix} \mathbf{f}_{it_{i1}}^\top \\ \vdots \\ \mathbf{f}_{it_{iT_i}}^\top \end{bmatrix}, \quad \boldsymbol{\epsilon}_i = \begin{bmatrix} \epsilon_{it_{i1}} \\ \vdots \\ \epsilon_{it_{iT_i}} \end{bmatrix}.$$

The model for the whole sample can be written in matrix form as

$$\mathbf{y} = \mathbf{X} \boldsymbol{\alpha} + \mathbf{P} \boldsymbol{\theta} + \mathbf{F} \boldsymbol{\psi} + \boldsymbol{\epsilon}, \quad (11)$$

$$\text{where } \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_N \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1_N \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{F}_1 \\ \vdots \\ \mathbf{F}_N \end{bmatrix}, \quad \boldsymbol{\epsilon} = \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \vdots \\ \boldsymbol{\epsilon}_N \end{bmatrix}$$

.....

and  $N^* = \sum_{i=1}^N T_i$  is the total number of observations.

## 7.2 Preconditioned conjugate gradient algorithm

Abowd et al. (1999) highlighted the challenges of fitting model (1) due to the large number of workers and firms. The US study contains  $N > 1$  million workers and  $J > 50,000$  firms. The Australian prototype dataset contains  $N > 10$  million workers and  $J > 1.5$  million firms for around 130 million observations over eleven years for the worker equation (1). This study uses the direct estimation methodology proposed by Abowd et al. (2002) which involves first solving a large sparse linear system with a preconditioned conjugate gradient algorithm, and then imposing constraints on the parameters to identify unique worker and firm effects. The conjugate gradient algorithm solves the sparse linear system  $\mathbf{A}\boldsymbol{\beta} = \mathbf{c}$ , where  $\mathbf{A}$  is a symmetric positive definite matrix,  $\boldsymbol{\beta}$  is an unknown vector and  $\mathbf{c}$  is a known vector. For ordinary least square estimation of parameters in (1), the system is defined with

$$\mathbf{A} = \begin{bmatrix} \mathbf{X}^\top \mathbf{X} & \mathbf{X}^\top \mathbf{P} & \mathbf{X}^\top \mathbf{F} \\ \mathbf{P}^\top \mathbf{X} & \mathbf{P}^\top \mathbf{P} & \mathbf{P}^\top \mathbf{F} \\ \mathbf{F}^\top \mathbf{X} & \mathbf{F}^\top \mathbf{P} & \mathbf{F}^\top \mathbf{F} \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\alpha} \\ \theta \\ \psi \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} \mathbf{X}^\top \mathbf{y} \\ \mathbf{P}^\top \mathbf{y} \\ \mathbf{F}^\top \mathbf{y} \end{bmatrix}. \quad (12)$$

Since  $\mathbf{A}$  is a large, sparse matrix, iterative methods like the conjugate gradient algorithm perform better if we transform  $\mathbf{A}$  to improve its condition number (Shewchuk, 1994). There are many options for creating a preconditioning matrix, including incomplete Cholesky factorisation or diagonal preconditioning which uses a diagonal matrix whose diagonal entries are identical to the diagonal elements of  $\mathbf{A}$  (see Song (2013) for a review). The preconditioning matrix used in the algorithm is a variant of incomplete Cholesky factorisation. Let

$$\mathbf{U} = \begin{bmatrix} \mathbf{Z} & 0 & 0 \\ 0 & \mathbf{P}^{1/2} & 0 \\ 0 & 0 & \mathbf{F}^{1/2} \end{bmatrix},$$

where  $\mathbf{Z}$  is the upper triangular matrix obtained from the Cholesky decomposition of  $\mathbf{X}^\top \mathbf{X}$ ,  $\mathbf{P}^{1/2}$  is the diagonal matrix with the square roots of the diagonal terms of  $\mathbf{P}^\top \mathbf{P}$  on the diagonal and  $\mathbf{F}^{1/2}$  is the diagonal matrix with the square roots of the diagonal terms of  $\mathbf{F}^\top \mathbf{F}$  on the diagonal. Following Fasshauer (2007), rewrite the system as

$$\tilde{\mathbf{A}}\tilde{\boldsymbol{\beta}} = \tilde{\mathbf{c}},$$

$$\text{where } \tilde{\mathbf{A}} = \mathbf{U}^{-\top} \mathbf{A} \mathbf{U}^\top = \begin{bmatrix} I & \mathbf{Z}^{-\top} \mathbf{X}^\top \mathbf{P} \mathbf{P}^{1/2} & \mathbf{Z}^{-\top} \mathbf{X}^\top \mathbf{F} \mathbf{F}^{1/2} \\ \mathbf{P}^{-1/2} \mathbf{P}^\top \mathbf{X} \mathbf{Z}^\top & I & \mathbf{P}^{-1/2} \mathbf{P}^\top \mathbf{F} \mathbf{F}^{1/2} \\ \mathbf{F}^{-1/2} \mathbf{F}^\top \mathbf{X} \mathbf{Z}^\top & \mathbf{F}^{-1/2} \mathbf{F}^\top \mathbf{P} \mathbf{P}^{1/2} & I \end{bmatrix}, \tilde{\boldsymbol{\beta}} = \mathbf{U}^{-\top} \boldsymbol{\beta} \text{ and } \tilde{\mathbf{c}} = \mathbf{U}^{-\top} \mathbf{c}.$$

The preconditioned conjugate gradient algorithm used in this study was developed by Dongarra (1991) and implemented in Fortran (see Algorithm 1). Let  $(k)$  denote the current and  $(k+1)$  the next iteration.

.....

The CG method computes  $\tilde{\beta}^{(k+1)}$  by iterating

$$\tilde{\beta}^{(k+1)} = \tilde{\beta}^{(k)} + \tilde{\alpha}^{(k)} \tilde{\mathbf{d}}^{(k)},$$

where  $\tilde{\alpha}^{(k)}$  is a scalar given by

$$\tilde{\alpha}^{(k)} = \frac{\tilde{\mathbf{r}}^{(k)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k)}}{\tilde{\mathbf{d}}^{(k)\top} \tilde{\mathbf{A}} \tilde{\mathbf{d}}^{(k)}}, \text{ with } \tilde{\mathbf{r}} = \tilde{\mathbf{c}} - \tilde{\mathbf{A}} \tilde{\beta}, \text{ and}$$

$$\tilde{\mathbf{d}}^{(k+1)} = \tilde{\mathbf{r}}^{(k+1)} + \tilde{\delta}^{(k+1)} \tilde{\mathbf{d}}^{(k)}, \text{ with } \tilde{\delta}^{(k+1)} = \frac{\tilde{\mathbf{r}}^{(k+1)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k+1)}}{\tilde{\mathbf{r}}^{(k)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k)}}.$$

The basic pseudo code is

---

**Algorithm 1** preconditioned conjugate gradient algorithm

---

- 1: **procedure**
  - 2:   compute the preconditioning matrix  $\mathbf{U}$
  - 3:   compute  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{c}}$
  - 4:   initial  $\mathbf{r}^{(0)} = \tilde{\mathbf{c}}$  and let  $\mathbf{d}^{(0)} = \mathbf{U}^{-1} \mathbf{r}^{(0)}$
  - 5:   for  $k = 1, 2, 3, \dots$  do
 
$$\tilde{\alpha}^k = \frac{\tilde{\mathbf{r}}^{(k)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k)}}{\tilde{\mathbf{d}}^{(k)\top} \tilde{\mathbf{A}} \tilde{\mathbf{d}}^{(k)}}$$

$$\tilde{\beta}^{(k+1)} = \tilde{\beta}^{(k)} + \tilde{\alpha}^k \tilde{\mathbf{d}}^{(k)}$$

$$\tilde{\mathbf{r}}^{(k+1)} = \tilde{\mathbf{r}}^{(k)} - \tilde{\alpha}^k \tilde{\mathbf{A}} \tilde{\mathbf{d}}^{(k)}$$

$$\tilde{\delta}^{(k)} = \frac{\tilde{\mathbf{r}}^{(k+1)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k+1)}}{\tilde{\mathbf{r}}^{(k)\top} \mathbf{U}^{-1} \tilde{\mathbf{r}}^{(k)}}$$

$$\tilde{\mathbf{d}}^{(k+1)} = \tilde{\mathbf{r}}^{(k+1)} + \tilde{\delta}^{(k+1)} \tilde{\mathbf{d}}^{(k)}$$
  - 6:   **until** the difference between  $\tilde{\beta}^{(k)}$  and  $\tilde{\beta}^{(k+1)}$  is less than  $10^{-7}$
  - 7:   **end procedure**
- 

The convergence criteria of  $\frac{|\tilde{\mathbf{r}}|}{|\tilde{\mathbf{c}}|} < 10^{-7}$  that we use is similar to that used by others, Abowd et al. (2002); Hallez et al. (2007)

### 7.3 Identification using grouping algorithm

The preconditioned CG algorithm does not provide a unique solution for the firm and worker effects. The solutions depend on the initial values, preconditioning matrices and convergence criteria and the implicit constraints used in the algorithm are not necessarily conveniently interpretable. The implicit constraints require the state equations to be satisfied at each iteration. Koopmans (1949), Koopmans et al. (1950) and Fisher (1966) discussed the need to impose model constraints to identify the underlying economic relationship in the observed data. This is because it is possible for two parametric equations to have the same likelihood function unless some restrictions are imposed to uniquely identify parameters. There are an infinite number of possible constraints and solutions. Fujikoshi (1993) summarises several possible approaches for two-way cross classified unbalanced data.



.....

### 7.3.1 Issues in Identification

We use a simplified version of model (1) in this subsection to illustrate the issues faced in imposing appropriate model restrictions on the model for workers' wages. For simplicity, we consider a single fixed  $t$  and replace the observable worker characteristics terms  $\mathbf{x}_{it}^\top \boldsymbol{\alpha}$  by the fixed unknown constant  $\mu$ . With these simplifications, the model (1) has expectation

$$E\{\ln(y_{it})\} = \mu + \theta_i + \mathbf{f}_{it}^\top \boldsymbol{\psi} = \mu + \theta_i + \psi_j, \tag{13}$$

when worker  $i$  works for firm  $j$  at time  $t$ . With  $t$  fixed, it is convenient to make the dependence on  $j$  more explicit and, just for this subsection, replace  $y_{it}$  by  $y_{ij}$ . We consider a two-way table of 5 workers labelled  $\theta_i$  for  $i = 1, \dots, 5$  and 4 firms labelled  $\psi_j$  for  $j = 1, \dots, 4$ . If we only have one observation in every cell, we can represent the table as shown in Figure 2. In practice, we often do not have one observation in every cell. A simple example is shown in Figure 3.

We describe the data in Figure 2 as balanced and in Figure 3 as unbalanced. The saturated model, the main effect without interaction model for the *balanced* data, is given by (13). The model matrix  $(\mathbf{P}, \mathbf{F})$  is given in Figure 1(1). The relationships between the columns in the model matrix in Figure 1(1) are

$$\beta_0 = \sum_{i=1}^5 \theta_i \tag{14a}$$

$$\beta_0 = \sum_{j=1}^4 \psi_j, \tag{14b}$$

where the sums are interpreted as the sums of the vectors in the columns labelled by  $\mu$ , the  $\theta_i$  and the  $\psi_j$ . These relationships show that the model is over-parameterised with ten parameters when only eight are needed so is rank deficient. This means that there are an infinite number of solutions that satisfy the ordinary least squares normal equation (1). The simplest way to identify unique solutions is by using the corner point constraint to set redundant parameters to zero, i.e.  $\theta_5 = \psi_4 = 0$  (Holmes et al., 1997). This is shown in Figure 1(2). After imposing the corner point constraint, the model is of full rank so the normal equations have a unique solution.

Figure 1: Model matrices for balanced two-way table

(1) no constraints										(2) with corner point constraints									
$\beta_0$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\psi_1$	$\psi_2$	$\psi_3$	$\psi_4$	$\beta_0$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\psi_1$	$\psi_2$	$\psi_3$		
1	1	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0		
1	1	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0		
1	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1		
1	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0		
1	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1	0	0		
1	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0		
1	0	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1		
1	0	1	0	0	0	0	0	0	1	1	0	1	0	0	0	0	1		
1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0		
1	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0		
1	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	1		
1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1		
1	0	0	0	1	0	1	0	0	0	1	0	0	0	1	1	0	0		
1	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0		
1	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1		
1	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0		
1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0		
1	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1	0		
1	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	1		
1	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0		

full  
rank

Figure 2: Balanced two-way table

	$\psi_1$	$\psi_2$	$\psi_3$	$\psi_4$
$\theta_1$	A	A	A	A
$\theta_2$	A	A	A	A
$\theta_3$	A	A	A	A
$\theta_4$	A	A	A	A
$\theta_5$	A	A	A	A

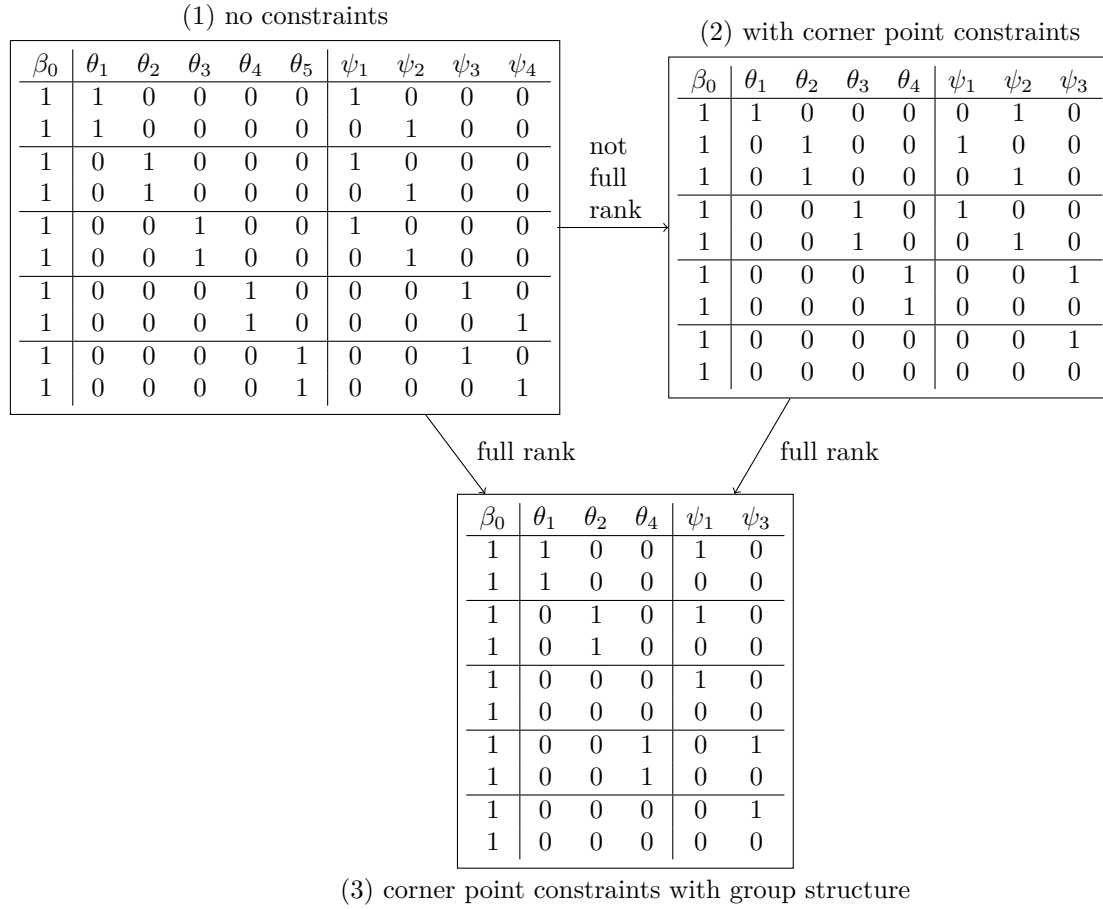
Figure 3: Unbalanced two-way table <sup>1</sup>

	$\psi_1$	$\psi_2$	$\psi_3$	$\psi_4$
$\theta_1$	A	A	NA	NA
$\theta_2$	A	A	NA	NA
$\theta_3$	A	A	NA	NA
$\theta_4$	NA	NA	A	A
$\theta_5$	NA	NA	A	A

In comparison, the model matrix for the unbalanced data is shown in Figure 4(1). As can be seen from Figure 3, the observation pattern forms two groups. This model is also rank deficient. If we apply corner point constraints by setting redundant parameters to zero, i.e.  $\theta_5 = \psi_4 = 0$ , the model matrix is shown in Figure 4(2).

<sup>1</sup>A = available NA = unavailable

Figure 4: Model matrices for unbalanced two-way table



However, the model matrix in Figure 4(2) is still singular because  $\theta_1 + \theta_2 + \theta_3 = \psi_1 + \psi_2$ .

$$\begin{array}{c}
 \theta_1 + \theta_2 + \theta_3 \\
 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
 \end{array}
 =
 \begin{array}{c}
 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0
 \end{array}
 \equiv
 \begin{array}{c}
 \psi_1 + \psi_2 \\
 \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{array}
 =
 \begin{array}{c}
 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0
 \end{array}$$

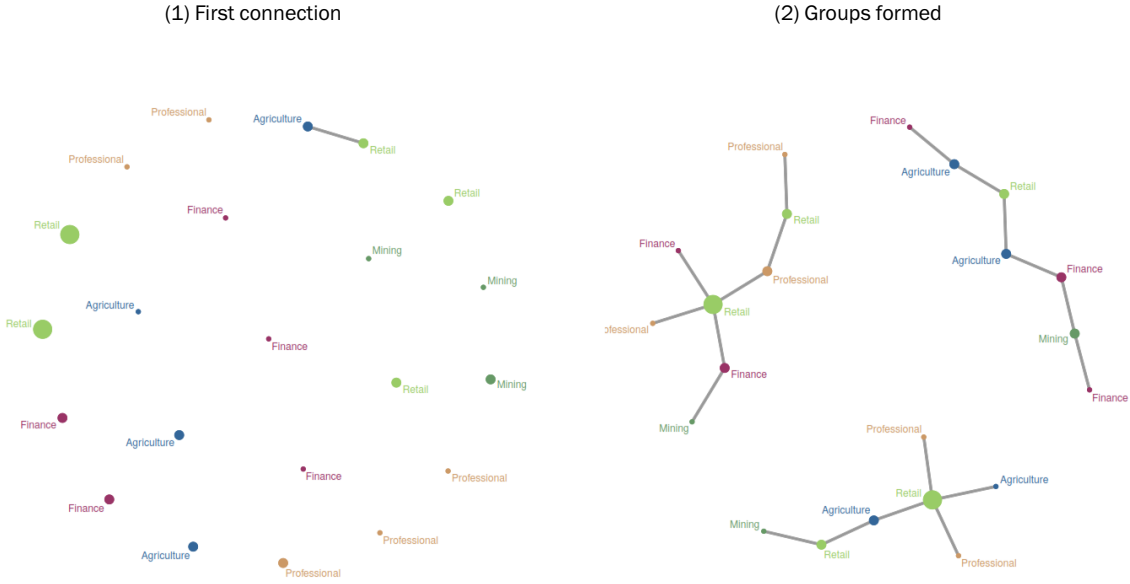
Figure 3 shows that the unbalanced data separates into two groups called connected groups (Searle, 1987). We need to take the grouping structure into account to identify unique firm and worker effects. There are an infinite number of possible constraints to make the model matrix of full rank; the particular choice from these is arbitrary. An example is to impose  $\psi_2 = 0$ . The model matrix for the resulting full rank model is shown in Figure 4(3).

Abowd et al. (2002) recognised the need to find connected groups of workers and firms to set model constraints to analyse linked employer and employee data. Firms and workers can be connected by a worker changing jobs or by multiple job holders who work for different firms. These connected groups are

.....

formed in such a way that no one worker or firm can be included in more than one group. Figure 5(1) and Figure 5(2) show how the algorithm connects firms and workers into mutually exclusive groups. The size of the circle represents the size of the firms to show that connections can occur between firms of different sizes. An edge connects two firms through a worker changing jobs from one firm to the other or holding jobs in both firms. These connected groups are mutually exclusive because there are no additional worker movements. See Algorithm 2 for details.

Figure 5: Connected groups



Abowd et al. (2002) proposed a grouping algorithm to create groups of connected workers and firms in the data for  $g = 1, \dots, G$  groups (see Algorithm 2).

---

**Algorithm 2** grouping algorithm

---

```
1: procedure
2:   Order by firm id and then worker id.
3:   for  $group = 1$ : assign first firm  $j$  to group  $g = 1$ ,
4:     partitioning step
5:     repeat
6:       add all workers employed by a firm  $j$  in group  $g = 1$  to group  $g = 1$ .
7:       add all firms that have employed a worker  $i$  in group  $g = 1$  to group  $g = 1$ .
8:     until no more firms or workers can be added to group  $g = 1$ .
9:     end partitioning step
10:  for  $group = 2$ :  $\forall$  worker  $i \notin g = 1$  and  $\forall$  firm  $j \notin g = 1$  assign first firm  $j$  to  $g = 2$ ,
    repeat partitioning step and add all workers and firms in group  $g = 2$  to group  $g = 2$ .
11:  for  $group = 3$ :  $\forall$  worker  $i \notin g = 1, 2$  and  $\forall$  firm  $j \notin g = 1, 2$  assign first firm  $j$  to  $g = 3$ ,
    repeat partitioning step and add all workers and all firms in group  $g = 3$  to group  $g = 3$ .
12:    :
13:  for  $group = G$ :  $\forall$  worker  $i \notin g = 1, 2, \dots, G-1$  and  $\forall$  firm  $j \notin g = 1, 2, \dots, G-1$  assign first firm  $j$  to  $g = G$ ,
    repeat partitioning step and add all workers and all firms in group  $g = G$  to group  $g = G$ .
14:  until all firms are assigned.
15:  end procedure
```

---

The algorithm divides connected workers and firms into mutually exclusive groups. A group is defined as all workers and firms that are connected through some migration of workers between firms in that group, and such that there is no migration of a worker within the group to any firm outside the group. The main result is that the ensuing model matrix is of full rank so the solutions to the ordinary least squares normal equations are unique.

## 8 EMPIRICAL RESULTS

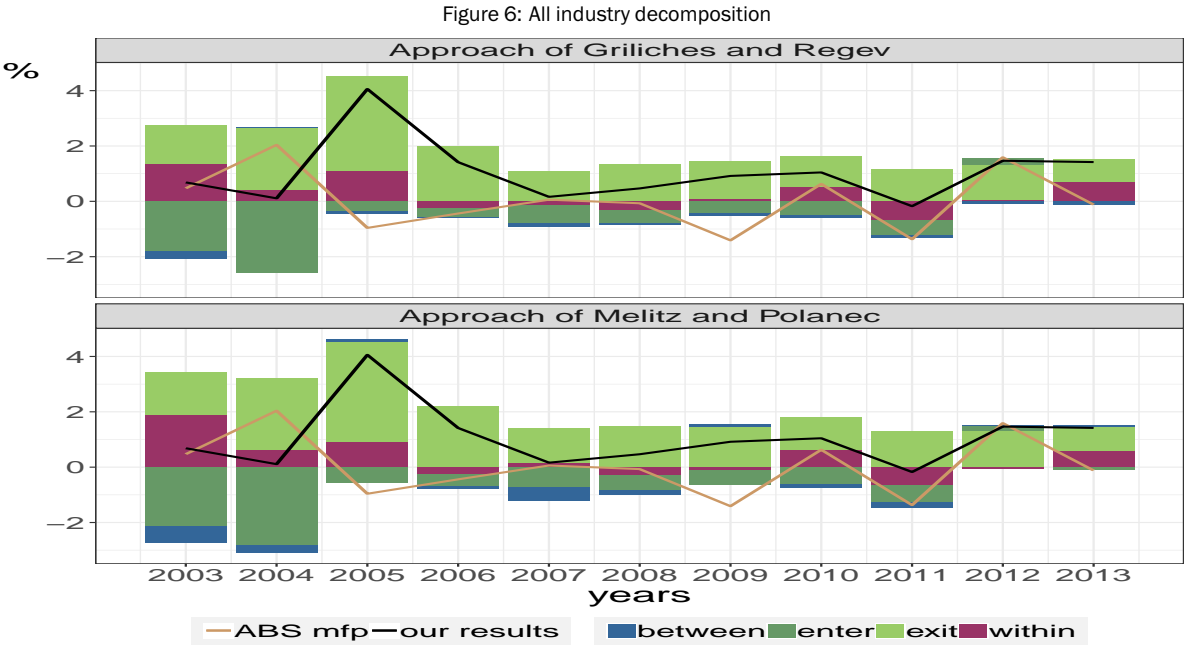
### 8.1 Firm dynamics and aggregate productivity

Figure 6 shows the estimated contributions from surviving, entering and exiting firms to aggregate productivity using Griliches and Regev (1995) and Melitz and Polanec (2015) methods. Nguyen and Hansell (2014) and Melitz and Polanec (2015) noted the importance of taking into account the appropriate counterfactual to derive the contributions from surviving, entering and exiting firms. We concur, particularly for the results from smaller industries as defined by gross value added (see Appendix A). The results show that the differences between the methods of Griliches and Regev (1995) and Melitz and Polanec (2015) are greater for entering and exiting firms in smaller industries. We have also explored the aggregation method proposed by Foster et al. (2001). The results are similar to the approach of Griliches and Regev (1995).

Figure 6 shows that our results are broadly consistent with published ABS annual productivity measures at the aggregate level. Our analysis provides useful insights into the variability in the firms' contributions to the aggregate productivity growth. This information is not available in the ABS publications. We find higher productivity growth over time in our results. The differences may be due to the fact that we use different prices to derive the volume measures. This introduces differences in relative prices when estimating firm productivity. These differences result in different substitution effects between labour and capital and between goods and services which can lead to different results. See Dumagan and Balk (2016) and Duarte and Restuccia (2017) on the role of relative prices in estimating productivity. In addition,

we use firm level capital cost instead of firm level capital stock measures for our analysis. This is because there is no information on the firm level asset prices. This information is required to derive capital stock measures using perpetual inventory method (Walters and Dippelsman, 1986).

This study shows the usefulness of firm level analysis to compare the contribution entering, exiting and surviving firms make to aggregate productivity. These contributions are quite different at the industry level. The analytical results can be extended to explore the link between share of younger firms to the overall growth to inform policies and encourage economic growth, see Andrews et al. (2015).



Like Nguyen and Hansell (2014), this study has found that the net contribution from entering and exiting firms is smaller in Manufacturing than in Services industries in general. The within industry contribution component generally has a smaller contribution in Services industries. This may imply that the entering and exiting firms are the main source of productivity changes.

8.2 Firm level model results

This study confirms the importance of correcting for endogeneity in estimating the firm-level production function. The estimated labour coefficients for the firm models *Wage* are higher than *L* in all industries (see Appendix B). The lower estimations of *L* are consistent with similar studies using instrumental variables to correct for the endogeneity. See Breunig and Wong (2008) and Levinsohn and Petrin (2003). The correction is important to avoid bias in the aggregate industry decomposition results.

8.3 Worker level model results

It is essential to include worker mobility between firms to uniquely identify the worker and firm effects (Abowd et al., 1999). Table 1 shows the pattern of mobility of workers in the sample. The columns indicate the number of jobs workers hold and the rows correspond to the number of job changes over the eleven years of data. There are significant worker movements in the sample. Only 23.27 per cent of workers hold a job without a change over the 11 year period.

Table 1: Number of job changes and number of jobs over ten years

no. of job changes	no. of jobs											Total
	1	2	3	4	5	6	7	8	9	10	>10	
0	23.27	.	.	.	.	.	.	.	.	.	.	23.27
1	14.22	4.60	.	.	.	.	.	.	.	.	.	18.82
2	8.73	4.52	1.31	.	.	.	.	.	.	.	.	14.56
3	5.45	3.79	1.49	0.38	.	.	.	.	.	.	.	11.11
4	3.47	2.91	1.44	0.44	0.12	.	.	.	.	.	.	8.37
5	2.24	2.14	1.24	0.46	0.13	0.04	.	.	.	.	.	6.24
6	1.44	1.54	1.01	0.43	0.14	0.04	0.02	.	.	.	.	4.63
7	0.93	1.09	0.79	0.38	0.14	0.04	0.01	0.01	.	.	.	3.40
8	0.61	0.76	0.61	0.31	0.13	0.04	0.02	0.01	.	.	.	2.48
9	0.39	0.53	0.45	0.26	0.11	0.04	0.02	0.01	.	.	.	1.81
10	0.26	0.37	0.34	0.21	0.10	0.04	0.02	0.01	.	.	.	1.34
>10	0.52	0.85	0.94	0.72	0.42	0.22	0.11	0.06	0.04	0.02	0.08	3.98
Total	61.53	23.10	9.62	3.59	1.29	0.46	0.20	0.10	0.04	0.02	0.08	100.01

Table 2 shows the correlation structure of the estimated components in the worker model. This study has found a positive correlation between the worker and firm effects. This is in line with the finding of Iranzo et al. (2008) but different from Abowd et al. (2002). Andrews et al. (2008) suggest that the negative correlation in previous studies may arise from a lack of worker mobility, which is not the case in this Australian sample.

Table 2: Pearson Correlation Coefficients of estimated components

<i>Prob &gt;  r </i> under $N = 130,281,096$					
	$\log L$	$\theta$	$\psi$	$\mathbf{X}\alpha$	$\epsilon$
$\log L$	1	0.3063***	0.54898***	-0.21151***	0.59226***
$\theta$	0.3063***	1	0.10584***	-0.97934***	-0.00845***
$\psi$	0.54898***	0.10584***	1	-0.09664***	-0.00206***
$\mathbf{X}\alpha$	-0.21151***	-0.97934***	-0.09664***	1	-0.02672***
$\epsilon$	0.59226***	-0.00845***	-0.00206***	-0.02672***	1

Note \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

---

## 9 CONCLUSIONS AND FUTURE DIRECTIONS

This study shows the value of using microdata to better understand the components of industry-level productivity growth. It explores methods for fitting a model for workers by solving a large sparse linear system of equations and uses the estimated results to correct for endogeneity in the firm's decisions about how much labour to employ. The paper also calculates the contribution of entering, exiting and surviving firms to aggregate productivity at the industry level.

Our results show the importance of correcting for endogeneity in estimating the production function. The productivity contributions from surviving, entering and exiting firms are quite different across different industries. Understanding these differences may be useful to inform policy.

Across all industries, we generally find that firm exit is the most important contributor to productivity growth. Firm entry generally has a negative impact on industry-level productivity growth. This is similar to what was found by Breunig and Wong (2008) for the 1990s in Australia. It is not surprising, as many new firms end up not surviving. They may lack access to industry-specific knowledge and skills.

Within-firm productivity increases are generally a positive contributor to industry-level productivity, but are very small in about half of the industry groups we examine. Re-allocation effects for continuing firms are virtually non-existent. Almost all of the reallocation is happening through entry and exit.

This would suggest that policies which facilitate firm entry and exit are likely to help in achieving increased productivity gains. Policies which provide large advantages to incumbent firms (such as cumbersome regulation which is difficult to comply with for new entrants) are likely to detract from productivity growth.

Our analysis could be extended in several ways. First, with a better proxy for worker skill such as education, we could better account for the effects of workers. Capturing workers' skill dispersion across and between firms would be useful. Secondly, it would be interesting to explore other estimation approaches like Constant Elasticity of Substitution production functions that allow the elasticity of substitution between capital and labour inputs to better understand the relative prices effects (McFadden, 1963) and (Steenkamp, 2017) . Increased data access and better measures of key variables are both required for such analyses.



.....

## 10 REFERENCES

- Abowd, J. M., Creecy, R. H., and Kramarz, F. (2002), “Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data,” Report, US Census Bureau, accessed at [link](#) on 12022016.
- Abowd, J. M., Kramarz, F., and Margolis, D. N. (1999), “High Wage Workers and High Wage Firms,” *Econometrica*, 67, 251–333, available [online](#).
- ABS (2009), “ANZSCO - Australian and New Zealand Standard Classification of Occupations,” 1220.0, available [online](#).
- (2015), “Information Paper: Construction of Experimental Statistics on Employee Earnings and Jobs from Administrative Data, Australia, 2011-12,” Available [online](#).
- (2018a), “5204.0 - Australian System of National Accounts, 2017-18,” Accessed at [link](#) on 01102018.
- (2018b), “6427.0 - Producer Price Indexes, Australia, Sep 2018,” Accessed at [link](#) on 01102018.
- ABS and DIIS (2017), “Business Longitudinal Analysis Data Environment,” Available [online](#).
- Andrews, D., Bartelsman, E., and Criscuolo, C. (2015), “Firm dynamics and productivity growth in Europe,” Report, mimeo.
- Andrews, M. J., Gill, L., Schank, T., and Upward, R. (2008), “High wage workers and low wage firms: negative assortative matching or limited mobility bias?” *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 171, 673–697.
- Bakhtiari, S. (2015), “Productivity, outsourcing and exit: the case of Australian manufacturing,” *Small Business Economics*, 44, 425–447.
- Bartelsman, E. J. and Dhrymes, P. J. (1998), “Productivity Dynamics: U.S. Manufacturing Plants, 1972–1986,” *Journal of Productivity Analysis*, 9, 5–34.
- Bartelsman, E. J. and Doms, M. (2000), “Understanding Productivity Lessons from Longitudinal Micro-data,” *Journal of Economic Literature*, 38, 569–594.
- Breunig, R. and Wong, M.-H. (2008), “A Richer Understanding of Australia’s Productivity Performance in the 1990s: Improved Estimates Based Upon Firm-Level Panel Data,” *Economic Record*, 84, 157–176.
- Chien, C.-H. and Mayer, A. (2015), “A New Analytical Platform to Explore Linked Data,” Report, Australian Bureau of Statistics, available [online](#).
- Dongarra, J. J. (1991), *Solving linear systems on vector and shared memory computers*, Philadelphia: Society for Industrial and Applied Mathematics.
- Duarte, M. and Restuccia, D. (2017), “Relative Prices and Sectoral Productivity,” Report, National Bureau of Economic Research.
- Dumagan, J. C. and Balk, B. M. (2016), “Dissecting aggregate output and labour productivity change: a postscript on the role of relative prices,” *Journal of Productivity Analysis*, 45, 117–119.
- Eberhardt, M. and Helmers, C. (2010), “Addressing transmission bias in micro production function models: A survey for practitioners,” Tech. rep., Working Paper, available [online](#).
- Fasshauer, G. (2007), “477 577 Numerical Linear Algebra Computational Mathematics I,” Available [online](#).

- .....
- Fisher, F. M. (1966), *The identification problem in econometrics*, McGraw-Hill.
- Foster, L., Haltiwanger, J. C., and Krizan, C. J. (2001), *Aggregate productivity growth: lessons from microeconomic evidence*, University of Chicago Press, pp. 303–372.
- Fox, J. T. and Smeets, V. (2011), “DOES INPUT QUALITY DRIVE MEASURED DIFFERENCES IN FIRM PRODUCTIVITY?” *International Economic Review*, 52, 961–989.
- Fujikoshi, Y. (1993), “Two-way ANOVA models with unbalanced data,” *Discrete Mathematics*, 116, 315–334.
- Gandhi, A., Navarro, S., and Rivers, D. A. (2011), “On the identification of production functions: How heterogeneous is productivity?” Available [online](#).
- Griliches, Z. and Regev, H. (1995), “Firm productivity in Israeli industry 1979–1988,” *Journal of Econometrics*, 65, 175–203.
- Hallez, H., Vanrumste, B., Grech, R., Muscat, J., De Clercq, W., Vergult, A., D’Asseler, Y., Camilleri, K. P., Fabri, S. G., and Van Huffel, S. (2007), “Review on solving the forward problem in EEG source analysis,” *Journal of neuroengineering and rehabilitation*, 4, 46.
- Holmes, A., Poline, J., and Friston, K. (1997), “Characterizing brain images with the general linear model,” in *Human Brain Function*, eds. Frackowiak, R., Friston, K., Frith, C., Dolan, R., and Mazziotta, J., Academic Press USA, pp. 59–84.
- Iranzo, S., Schivardi, F., and Tosetti, E. (2008), “Skill Dispersion and Firm Productivity: An Analysis with Matched Employer-Employee Data,” *Journal of Labor Economics*, 26, 247–285.
- Koopmans, T. C. (1949), “Identification problems in economic model construction,” *Econometrica, Journal of the Econometric Society*, 125–144.
- Koopmans, T. C., Rubin, H., and Leipnik, R. B. (1950), *Measuring the equation systems of dynamic economics*, New York: John Wiley and Sons, vol. Monograph No. 10, pp. 53–237.
- Lafrance, A. and Baldwin, J. R. (2011), “Firm Turnover and Productivity Growth in Selected Canadian Services Industries, 2000 to 2007,” Tech. rep., Statistics Canada, Analytical Studies Branch.
- Lentz, R. and Mortensen, D. T. (2010), “Labor market models of worker and firm heterogeneity,” *Annu. Rev. Econ.*, 2, 577–602.
- Levinsohn, J. and Petrin, A. (2003), “Estimating Production Functions Using Inputs to Control for Unobservables,” *The Review of Economic Studies*, 70, 317–341.
- Mare, D. C., Hyslop, D. R., and Fabling, R. (2016), “Firm productivity growth and skill,” *New Zealand Economic Papers*, 1–25.
- McFadden, D. (1963), “Constant elasticity of substitution production functions,” *The Review of Economic Studies*, 30, 73–83.
- Melitz, M. J. and Polanec, S. (2015), “Dynamic Olley-Pakes productivity decomposition with entry and exit,” *The RAND Journal of Economics*, 46, 362–375.
- Nguyen, T. and Hansell, D. (2014), “Firm dynamics and productivity growth in Australian manufacturing and business services Oct 2014,” Report, ABS, available [online](#).
- .....

- .....
- Olley, G. S. and Pakes, A. (1996), “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 64, 1263–1297.
- Reed, W. R. (2015), “On the Practice of Lagging Variables to Avoid Simultaneity,” *Oxford Bulletin of Economics and Statistics*, 77, 897–905.
- Searle, S. R. (1987), *Linear models for unbalanced data*, New York: Wiley.
- Shewchuk, J. R. (1994), “An introduction to the conjugate gradient method without the agonizing pain,” Accessed at [link](#) on 2017-08-03.
- Song, H. (2013), “ECS231 Large-Scale Scientific Computation Course College of Engineering,” Available [online](#).
- Steenkamp, D. (2017), “Productivity and relative prices,” Thesis, accessed at [link](#) on 2018-08-03.
- Syverson, C. (2011), “What Determines Productivity?” *Journal of Economic Literature*, 49, 326–365.
- Walters, R. and Dippelsman, R. J. (1986), *Estimates of depreciation and capital stock, Australia*, Australian Bureau of Statistics, available [online](#).
- Wilkinson, G. N. and Rogers, C. E. (1973), “Symbolic Description of Factorial Models for Analysis of Variance,” *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 22, 392–399.
- Zellner, A., Kmenta, J., Dr, xe, and ze, J. (1966), “Specification and Estimation of Cobb-Douglas Production Function Models,” *Econometrica*, 34, 784–795.

A INDUSTRY DECOMPOSITION

Figure 7: Industry Decomposition

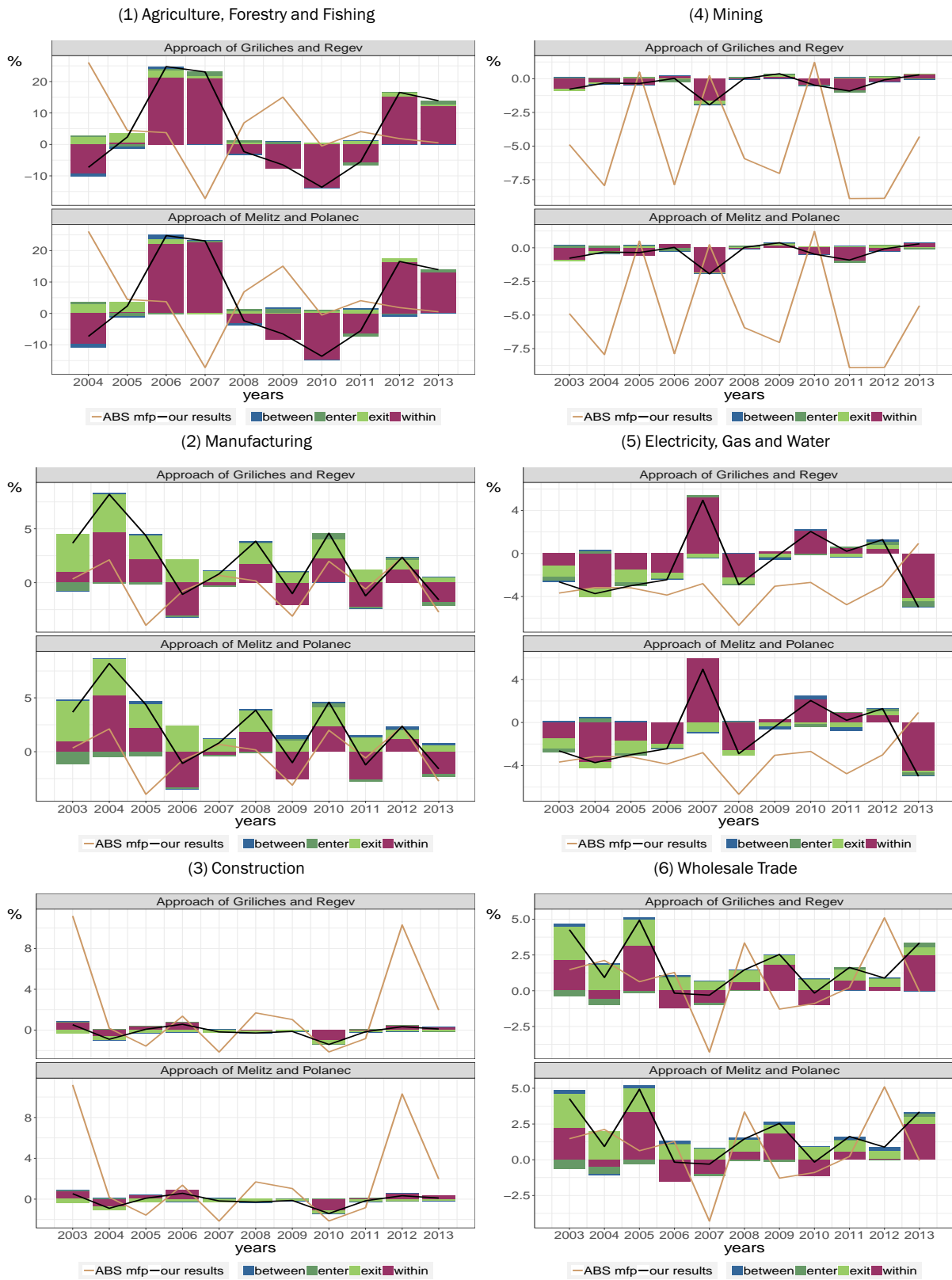


Figure 8: Industry Decomposition



Figure 9: Industry Decomposition



B FIRM MODEL RESULTS

	ALL.IV (1)	ALL (2)	A.IV (3)	B.IV (4)	C.IV (5)	A (6)	B (7)	C (8)
LL_D	0.431*** (0.001)		0.345*** (0.002)	-0.878*** (0.018)	0.394*** (0.003)			
WAGES		0.695*** (0.001)				0.480*** (0.002)	-0.121*** (0.019)	0.791*** (0.003)
LEK	0.095*** (0.0004)	0.042*** (0.0004)	0.283*** (0.002)	-0.352*** (0.013)	0.031*** (0.002)	0.242*** (0.002)	-0.349*** (0.013)	-0.027*** (0.002)
LEM	0.066*** (0.0003)	-0.003*** (0.0004)	-0.143*** (0.002)	-0.090*** (0.009)	0.200*** (0.002)	-0.176*** (0.002)	-0.176*** (0.010)	0.061*** (0.002)
FIRM_AGE	0.028*** (0.0002)	0.031*** (0.0002)	0.006*** (0.001)	0.074*** (0.004)	0.039*** (0.001)	0.013*** (0.001)	0.074*** (0.004)	0.034*** (0.001)
year_2004	-0.034*** (0.003)	-0.020*** (0.003)	-0.070*** (0.012)	-0.096 (0.079)	-0.057*** (0.011)	-0.062*** (0.012)	-0.105 (0.080)	-0.035*** (0.010)
year_2013	-0.135*** (0.003)	-0.049*** (0.003)	-0.181*** (0.014)	-0.507*** (0.076)	-0.124*** (0.011)	-0.127*** (0.014)	-0.608*** (0.076)	0.025** (0.011)
ind_A	-0.393*** (0.003)	-0.251*** (0.003)						
ind_R	0.188*** (0.005)	0.132*** (0.005)						
Constant	-0.715*** (0.006)	0.806*** (0.004)	-0.142*** (0.019)	13.780*** (0.202)	-0.557*** (0.029)	1.382*** (0.012)	5.232*** (0.111)	0.517*** (0.016)
Observations	38,176,012	38,176,012	2,889,066	117,857	1,526,089	2,889,066	117,857	1,526,089
Adjusted R <sup>2</sup>	0.051	0.070	0.020	0.036	0.034	0.027	0.017	0.060

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

---



---

	IV.D	IV.E	IV.F	IV.G	D	E	F	G
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LL_D	0.139*** (0.012)	0.285*** (0.001)	0.445*** (0.004)	0.383*** (0.003)				
WAGES_WPI					0.477*** (0.013)	0.680*** (0.001)	0.907*** (0.005)	0.890*** (0.003)
LEK	0.291*** (0.009)	0.153*** (0.001)	0.046*** (0.004)	-0.007*** (0.002)	0.224*** (0.009)	0.114*** (0.001)	-0.044*** (0.004)	-0.056*** (0.002)
LEM	-0.005 (0.007)	-0.018*** (0.001)	0.072*** (0.003)	0.240*** (0.002)	-0.056*** (0.007)	-0.106*** (0.001)	-0.052*** (0.003)	0.071*** (0.002)
FIRM_AGE	0.051*** (0.003)	-0.001* (0.0004)	0.099*** (0.001)	0.099*** (0.001)	0.046*** (0.003)	-0.003*** (0.0004)	0.094*** (0.001)	0.086*** (0.001)
year_2004	-0.114** (0.054)	-0.012** (0.006)	-0.048*** (0.018)	-0.102*** (0.011)	-0.115** (0.054)	-0.003 (0.005)	-0.021 (0.018)	-0.066*** (0.011)
year_2013	-0.231*** (0.051)	-0.032*** (0.006)	-0.273*** (0.018)	-0.264*** (0.011)	-0.188*** (0.051)	0.030*** (0.006)	-0.121*** (0.017)	-0.083*** (0.011)
Constant	2.241*** (0.129)	1.850*** (0.014)	-1.507*** (0.046)	-1.371*** (0.028)	1.776*** (0.074)	2.010*** (0.008)	-0.358*** (0.026)	-0.456*** (0.017)
Observations	86,888	6,382,179	1,207,357	2,288,186	86,888	6,382,179	1,207,357	2,288,186
Adjusted R <sup>2</sup>	0.019	0.012	0.025	0.035	0.032	0.039	0.044	0.062

---



---



	H.IV	I.IV	J.IV	K.IV	H	I	J	K
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LL_D	0.264*** (0.003)	0.300*** (0.002)	0.630*** (0.005)	0.561*** (0.002)				
WAGES_WPI					0.734*** (0.003)	0.632*** (0.002)	1.001*** (0.006)	0.526*** (0.002)
LEK	-0.004* (0.002)	0.233*** (0.001)	0.067*** (0.005)	-0.015*** (0.001)	-0.056*** (0.002)	0.165*** (0.001)	-0.007 (0.005)	-0.064*** (0.001)
LEM	0.509*** (0.002)	0.038*** (0.001)	0.127*** (0.004)	0.016*** (0.001)	0.336*** (0.002)	-0.009*** (0.001)	0.018*** (0.004)	-0.015*** (0.001)
FIRM_AGE	0.084*** (0.001)	0.010*** (0.001)	0.074*** (0.002)	0.042*** (0.001)	0.066*** (0.001)	0.006*** (0.001)	0.081*** (0.002)	0.059*** (0.001)
year_2004	-0.109*** (0.011)	-0.009 (0.008)	-0.043 (0.030)	0.013 (0.008)	-0.080*** (0.010)	-0.0002 (0.007)	-0.009 (0.030)	0.014* (0.008)
year_2013	-0.301*** (0.010)	-0.046*** (0.008)	-0.074** (0.029)	-0.023*** (0.008)	-0.123*** (0.010)	0.008 (0.008)	0.044 (0.029)	-0.005 (0.008)
Constant	-0.912*** (0.025)	1.069*** (0.018)	-3.304*** (0.056)	-1.633*** (0.018)	-0.487*** (0.015)	1.529*** (0.011)	-1.148*** (0.037)	1.841*** (0.010)
Observations	1,294,745	2,302,966	335,094	5,093,293	1,294,745	2,302,966	335,094	5,093,293
Adjusted R <sup>2</sup>	0.073	0.036	0.065	0.025	0.108	0.066	0.091	0.022

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

	IV.L	IV.M	IV.N	IV.O	L	M	N	O
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LL_D	0.445*** (0.002)	0.481*** (0.001)	0.337*** (0.002)	-0.089*** (0.008)				
WAGES_WPI					0.782*** (0.002)	0.816*** (0.001)	0.730*** (0.002)	0.083*** (0.009)
LEK	0.165*** (0.001)	0.016*** (0.001)	0.087*** (0.002)	0.176*** (0.005)	0.116*** (0.001)	-0.033*** (0.001)	0.025*** (0.002)	0.173*** (0.005)
LEM	0.064*** (0.001)	0.154*** (0.001)	0.019*** (0.001)	0.156*** (0.006)	-0.008*** (0.001)	0.058*** (0.001)	-0.050*** (0.001)	0.115*** (0.006)
FIRM_AGE	0.027*** (0.001)	0.001*** (0.0004)	0.006*** (0.001)	0.060*** (0.003)	0.032*** (0.001)	0.007*** (0.0003)	0.003*** (0.001)	0.055*** (0.003)
year_2004	-0.024*** (0.008)	-0.018*** (0.006)	-0.038*** (0.010)	-0.090*** (0.035)	-0.019** (0.008)	0.002 (0.006)	-0.026*** (0.010)	-0.089*** (0.035)
year_2013	-0.068*** (0.009)	-0.084*** (0.006)	-0.100*** (0.010)	-0.122*** (0.038)	-0.024*** (0.008)	0.009 (0.006)	-0.019** (0.010)	-0.117*** (0.038)
Constant	-0.948*** (0.019)	-0.585*** (0.014)	0.987*** (0.020)	3.697*** (0.076)	0.248*** (0.011)	0.789*** (0.008)	1.331*** (0.011)	2.588*** (0.045)
Observations	3,498,223	4,407,977	1,563,592	202,726	3,498,223	4,407,977	1,563,592	202,726
Adjusted R <sup>2</sup>	0.031	0.049	0.028	0.026	0.062	0.098	0.075	0.026

	IV.P	IV.Q	IV.R	IV.S	P	Q	R	S
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LL_D	1.290*** (0.004)	0.419*** (0.002)	0.450*** (0.003)	0.458*** (0.003)				
WAGES_WPI					1.526*** (0.004)	0.657*** (0.003)	0.796*** (0.004)	0.793*** (0.003)
LEK	-0.073*** (0.004)	0.107*** (0.002)	-0.035*** (0.003)	0.082*** (0.002)	-0.140*** (0.004)	0.084*** (0.002)	-0.084*** (0.003)	0.051*** (0.002)
LEM	0.218*** (0.003)	0.140*** (0.001)	0.128*** (0.002)	0.379*** (0.002)	0.180*** (0.003)	0.091*** (0.001)	0.069*** (0.002)	0.279*** (0.002)
FIRM_AGE	-0.022*** (0.001)	0.010*** (0.001)	0.042*** (0.001)	0.005*** (0.001)	-0.012*** (0.001)	0.003*** (0.001)	0.043*** (0.001)	0.005*** (0.001)
year_2004	0.105*** (0.021)	-0.006 (0.011)	-0.044** (0.017)	-0.028*** (0.010)	0.145*** (0.021)	0.014 (0.011)	-0.036** (0.017)	-0.005 (0.010)
year_2013	-0.102*** (0.021)	0.015 (0.011)	-0.113*** (0.019)	-0.131*** (0.011)	0.096*** (0.021)	0.143*** (0.011)	-0.039** (0.018)	-0.002 (0.010)
Constant	-9.630*** (0.036)	-0.427*** (0.023)	-0.583*** (0.031)	-2.412*** (0.026)	-3.870*** (0.022)	0.799*** (0.015)	0.583*** (0.019)	-1.034*** (0.015)
Observations	603,115	1,617,060	747,311	2,012,288	603,115	1,617,060	747,311	2,012,288
Adjusted R <sup>2</sup>	0.219	0.045	0.044	0.066	0.225	0.063	0.076	0.086

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

C SUMMARY STATISTICS

Table 3: Summary Statistics - Firm Level Productivity Model

Statistic	N	Mean	St. Dev.	Min	Max
FIRM_AGE	38,176,012	5.742	3.992	1	20
LL_D	38,176,012	10.360	1.171	-0.857	18.620
LEY_RS	38,176,012	4.774	3.477	-44.2	31.88
LEK	38,176,012	3.853	1.340	-88.390	16.420
LEM	38,176,012	5.127	1.872	-69.220	37.790
WAGES_WPI	38,176,012	4.999	1.092	-9.588	16.230
ABN year	38,176,012	2008		2002	2013

<sup>1</sup> FIRM\_AGE is the logarithm of firm age. Firm age is derived as the current year minus the year of incorporation.

<sup>2</sup> LL\_D the logarithm of labour inputs.

<sup>3</sup> LEY\_RS is logarithm of per employee value added (i.e. sales adjusted for repurchase of stock) deflated by industry Gross Value Added implicit price deflators

<sup>4</sup> LEK is the logarithm of per employee cost of capital that includes depreciation, capital rental expenses and capital work deductions deflated by the industry consumption of fixed capital implicit price deflators.

<sup>5</sup> LEM is logarithm of per employee material costs deflated by Producer Price Indexes Intermediate Goods.

<sup>6</sup> WAGES\_WPI is the logarithm of per employee wage costs (reported in Business Activities Statements) deflated by Wage Price Index - All Industries.

Table 4: Summary Statistics Worker Equation

Statistic	N	Mean	St. Dev.	Min	Max
SKILLH	130,281,096	0	0	0.31	1
SKILLHM	130,281,096	0	0	0.11	1
SKILLM	130,281,096	0	0	0.12	1
yr2003	130,281,096	0	0	0.07	1
yr2004	130,281,096	0	0	0.07	1
yr2005	130,281,096	0	0	0.07	1
yr2006	130,281,096	0	0	0.07	1
yr2007	130,281,096	0	0	0.08	1
yr2008	130,281,096	0	0	0.08	1
yr2009	130,281,096	0	0	0.08	1
yr2010	130,281,096	0	0	0.12	1
yr2011	130,281,096	0	0	0.11	1
yr2012	130,281,096	0	0	0.10	1
yr2013	130,281,096	0	0	0.09	1
AGE	130,281,096	17	37	37	64
AGE2	130,281,096	289	1369	1549	4096
AGE3	130,281,096	4913	50653	70029	262144
AGE4	130,281,096	83521	1874161	3370501	16777216
SEXAGE	130,281,096	0	18	19	64
SEXAGE2	130,281,096	0	324	792	4096
SEXAGE3	130,281,096	0	5832	35825	262144
SEXAGE4	130,281,096	0	104976	1726528	16777216
SEX2003	130,281,096	0	0	0.03	1
SEX2004	130,281,096	0	0	0.04	1
SEX2005	130,281,096	0	0	0.04	1
SEX2006	130,281,096	0	0	0.04	1
SEX2007	130,281,096	0	0	0.04	1
SEX2008	130,281,096	0	0	0.04	1
SEX2009	130,281,096	0	0	0.04	1
SEX2010	130,281,096	0	0	0.06	1
SEX2011	130,281,096	0	0	0.06	1
SEX2012	130,281,096	0	0	0.05	1
SEX2013	130,281,096	0	0	0.05	1

<sup>1</sup> The indicator variable HighSkill (SKILLH) equals 1 if a worker has at least tertiary qualification and 0 otherwise.

<sup>2</sup> The indicator variable MediumSkill (SKILLHM) equals 1 if a worker has at most a diploma qualification and 0 otherwise.

<sup>3</sup> The indicator variable WorkingSkill (SKILLM) equals 1 if a worker has at most a certificate III qualification and 0 otherwise.

<sup>4</sup> The Time indicator variables, yr2003-yr2013, are eleven years time indicator variables. One for each year between 2003 to 2013.

<sup>5</sup> AGE is the logarithm of worker age. Worker age is derived as the current year minus the year of birth. AGE2, AGE3 and AGE4 are worker AGE in quadratic, cubic and quartic.

<sup>6</sup> SEXAGE, SEXAGE2, SEXAGE3 and SEXAGE4 are the interaction terms between SEX and polynomial AGE.

<sup>7</sup> SEX2003-SEX2013 are the interaction terms between SEX and Time indicator variables.

---

## 11 ACKNOWLEDGEMENTS

Authors would like to express our gratitude to the following ABS colleagues - Lisette Aaron, Michael Beahan, Diane Braskic, Stephen Cross, Dr Philip Gould, Rowan Hatley, Dr Sarah Hinde, Jennifer Humphrys, Jacqui Jones, Grace Kim, Katherine Keenan, Lachlan McColl, Janelle McFarlane, Sybille McKeown, Katrina Richardson, Paul Roberts, Helen Robson, Dr Siu-Ming Tam, David Taylor, Liza Tiy, Mark Viney, Hui Wei and Carter Wong for their helpful comments and support for this research, Dr David Maré and Cornell University Labor Dynamics Institute for making Fortran code available, and those who provided comments at the Business Analytic and Econometrics session of the Australian Statistical Conference 2016 that helped improve our paper. We remain solely responsible for the views expressed in this paper.

## FOR MORE INFORMATION . . .

**www.abs.gov.au** the ABS website is the best place for data from our publications and information about the ABS.

### INFORMATION AND REFERRAL SERVICE

Our consultants can help you access the full range of information published by the ABS that is available free of charge from our website. Information tailored to your needs can also be requested as a 'user pays' service. Specialists are on hand to help you with analytical or methodological advice.

POST Client Services, ABS, GPO Box 796, Sydney NSW 2001

FAX 1300 135 211

EMAIL [client.services@abs.gov.au](mailto:client.services@abs.gov.au)

PHONE 1300 135 070

## FREE ACCESS TO STATISTICS

All ABS statistics can be downloaded free of charge from the ABS web site.

**WEB ADDRESS** [www.abs.gov.au](http://www.abs.gov.au)