

EMBARGO: 11:30AM (CANBERRA TIME) 10 DECEMBER 1999

Working Paper No. 99/2

# SEASONAL ADJUSTMENT

## Comparison of Philosophies

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December 1999

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ABS Catalogue No. 1351.0

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Produced by the Australian Bureau of Statistics

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## Abstract

This paper compares the theory and practical use of two major seasonal adjustment philosophies. They are based on a direct filter approach to time series decomposition made popular by X11 and the model based approach.

**Keywords:** X11, model based, spectrum, gain, TRAMO/SEATS

# SEASONAL ADJUSTMENT - COMPARISON OF PHILOSOPHIES.

## 1. Introduction

Until recently, most national statistical agencies used the X-11 method (developed by Shiskin et al at the US Bureau of the Census) or enhanced versions of X-11 for seasonal adjustment of their time series. In recent years, some competing methods have emerged.

Most seasonal adjustment is based on either of two broad procedures:

- Applying filters to split the time series into its trend, seasonal and irregular components. Typically, symmetric linear filters are applied to the middle or interior part of the series, and asymmetric linear filters are applied to the ends of the series. Methods in this group which we shall call “X-11 style” include X-11, X-11ARIMA88, X12-ARIMA, STL, SABL and SEASABS. Organisations using an “X-11 style” adjustment include the US Bureau of the Census, Statistics Canada, Office of National Statistics UK, Australian Bureau of Statistics and Statistics New Zealand.
- Modelling the time series. This procedure can entail either modelling the trend, seasonal and irregular components of the series or modelling the original series and, from that model, computing the components. Methods in this group which we shall call “model-based” include STAMP and TRAMO/SEATS. Eurostat is a major user of model-based adjustment.

This paper compares these two procedures (or “philosophies”). It shows how either can be justified as a valid approach to seasonal adjustment. The distinctive features of each philosophy are emphasised, as are areas where problems can occur. Section 1 gives a broad outline of the seasonal adjustment problem and details of the two philosophies to solving it. Section 2 compares these philosophies in a theoretical and practical way. Section 3 discusses the X-11 style methods and section 4 the model-based methods. Section 5 makes some final comments and suggestions regarding where future advances to seasonal adjustment may be made.

### 1.1 The aims of seasonal adjustment

The origin of the seasonal adjustment problem is the undisputed fact that most quarterly and monthly economic time series have a strong seasonal component. In virtually all economic data, the seasonality is changing over time. This component makes interpretation of the data by users without sophisticated tools very difficult. The aim of seasonal adjustment is therefore to remove the seasonal component.

An exact definition of seasonal adjustment is impossible, since the seasonal component of real-world data is unobserved. Moreover, the phrase “seasonal adjustment” has been widened over the years to include the removal of other fairly well defined time series components such as trading day and moving holidays. The broader phrase “adjusted data” is a more accurate description of what the adjustment process actually delivers to the users of economic data. A key question is how far can the meaning of “adjusted” be stretched -- this point is expanded upon later in the paper. Another key point is how far the “automatic options” provided by seasonal adjustment packages can yield sensible adjustments and when the intervention of human experts is required.

## **1.2 X-11 style seasonal adjustment**

Generally the philosophy of X-11 is to remove a band of spectral power around the seasonal harmonics: these harmonics occur at 12, 6, 4, 3, 2.4 and 2 cycles per year in a monthly time series. The width of this band varies according to the characteristics of the data, but the band is always limited to “near” the seasonal harmonics regardless of the spectrum of the original data. In addition, the trend is always required to be “smooth”. Given such criteria, it is obvious that the irregular component does not necessarily have white noise characteristics.

These features of the adjustment process derive directly from the properties of the filters used, and do not require recourse to any explicit model of the components apart from the “rules” regarding spectral bands and smoothness of trends. X-11 style methods are generally of an iterative nature -- the trend, seasonal and irregular components are estimated separately in a computation loop. For example, in X-11 a first guess at the trend is computed using a 2x12 moving average. The original data is then detrended to give an estimate of the combined seasonal-and-irregular component. This is then smoothed month-by-month using (for a standard X-11 run) a 3x5 moving average to yield a separate estimate of the seasonal. The computations continue through many iterations until convergence criteria are satisfied.

Major computational differences between the various methods in the X-11 family are usually due to different techniques used at the ends of the time series. For example, some methods use asymmetric filters at the ends, while others extend the series using ARIMA models and apply symmetric filters to the extended series. Section 3 provides more details of X-11 style seasonal adjustments.

## **1.3 Model-based seasonal adjustment**

Generally the philosophy of model-based adjustment is to provide models for the components; the irregular component is often required to be white noise. The component models are usually estimated simultaneously using the Kalman filter or allied techniques. Each component is assigned its own model and, in particular, its own innovations variance. The model determines the broad spectral characteristics of each component; the innovations variance determines the relative strength of each component. The contributions of the components are usually estimated using maximum likelihood methods.

Major computational differences between the various methods in the model-based family are usually due to model specification. In some cases, the components are modelled directly; in others, the original data series is modelled and the component models computed from that model. More details of model-based seasonal adjustment are provided in section 4.

## 2 Comparison of Philosophies

### 2.1 Theory

We illustrate the differences between the X-11 style and model-based adjustments using data generated by the Box and Jenkins (1976) “airline model”. This model has been used to describe a wide range of economic time series, and is often used to illustrate the features of time series analysis methods. A model with parameters close to those used for the original airline data is given by:

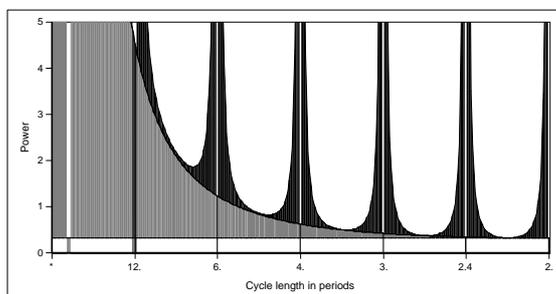
$$2.1 \quad (1 - B)(1 - B^{12})O_t = (1 - 0.5117B)(1 - 0.6845B^{12})EO_t$$

The two competing approaches can be demonstrated for this model graphically - see GRAPH 1 below. The black part of the total spectrum is the seasonal component, the grey the trend and the white the irregular.

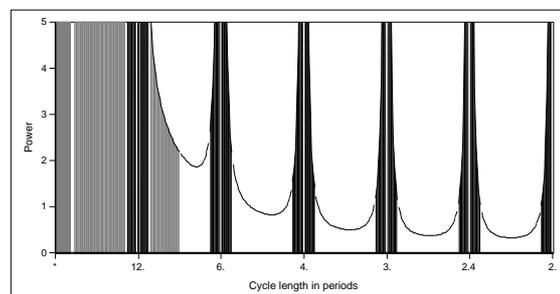
- For a model-based analysis, the irregular is usually defined as white noise, the trend has a "random walk" look and the seasonal is the remainder.
- In the X-11 style analysis, the seasonal is defined as a band around the seasonal harmonics, the trend is defined as cycles longer than a certain length and the irregular is the remainder.

GRAPH 1 SEASONAL ADJUSTMENT

1a. Model based solution



1b. "X-11 style" solution



Among the differences between the two competing approaches are the following:

First, the two approaches use a different order when specifying the time series components. Under the model-based approach, the irregular component is typically specified first. This is particularly true of packages like TRAMO/SEATS where the irregular is assumed to have the largest variance possible given the spectrum for the original. This can be seen from GRAPH 1a where the white noise irregular is a horizontal block of maximum height. In the X-11 style approach, the irregular is defined last as the residual after the seasonal and trend are defined.

Second, the X-11 approach aims to remove all spectral power at the seasonal frequencies whereas the model-based approach theoretically leaves some power at the seasonal frequencies in the white noise of the irregular component and also in the trend (consistent with the model specifications for these components).

Third, under the model-based approach the mix of trend, seasonal and irregular for a given model is defined by the variance of the innovations for each component model, often called the “hyper parameters”. In X-11, the size of the bands around the seasonal harmonics are determined by the seasonal filter length, and the frequency cut off for the trend is determined by the length of the trend filter.

For this type of data (with strong trend and seasonal components), there are many plausible adjustments matching perfectly defensible definitions of seasonality. In a case like that above, provided that the X-11 style adjustment exploits the “known” model (as is done by the popular X11-ARIMA and X12-ARIMA variants) to obtain the end-series estimates, the X-11 solution to the adjustment problem is just as valid as the model-based solution. Moreover, the X-11 solution may be preferable for an official statistical agency due to the simplicity of removing a band around the seasonal harmonics for all data (and the simplicity of explaining to users what has been done to the data). Of course, in practice ARIMA models used to extend the time series only approximate what is clearly a complex non-linear process of real economic data. In practice the “real” model is unknown and always will be.

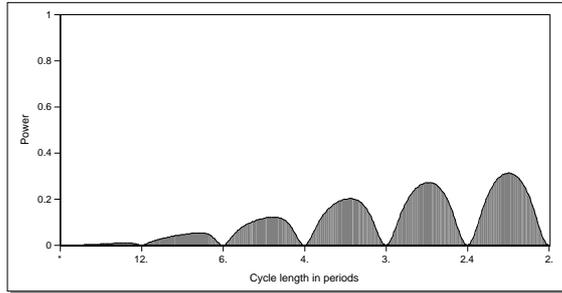
## **2.2 Practice**

Section 2.1, discussed some of the theoretical differences between the two philosophies. In practice, the adjustments shown in GRAPH 1 cannot be obtained exactly, only approximated for finite amounts of real-world data. This is true for both the model-based and X-11 style methods.

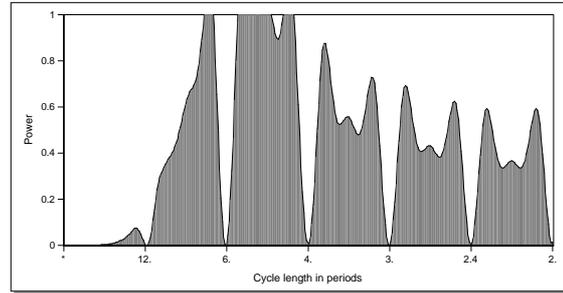
For the model-based adjustments, the components are typically estimated using a Minimum Mean Square Error criterion (MMSE). This is discussed in more detail later in the paper, but the main result is that the decomposition implied by the component models cannot be obtained exactly but only approximately. When the X-11 style adjustment is applied to finite amounts of data, the exact or “cliff” cutoffs shown in GRAPH 1b are not obtainable, one observes gradual or “slope” cutoffs in practice. GRAPH 2 shows the spectra of the irregular allowing for these estimation problems.

## GRAPH 2 IRREGULAR ESTIMATED SPECTRUM

2a. Model based



2b. X-11 based



The following points may be noted when one compares the theoretical and empirical spectra:

- **Model-based.** Compared to the theoretical decomposition, the estimated decomposition for the model-based irregular has dips at the trend and seasonal harmonics and looks nothing like the horizontal line suggested by theory. In the case of the airline data, these dips are the price of having to estimate powerful trend and seasonal components.
- **X-11 style.** In the case of the X-11 style adjustment, the cut off between the irregular and trend is a slope rather than an exact cut off -- similarly, the seasonal bands are delineated by slopes rather than by cliffs. There are also strange “wiggles”, caused by certain imperfections in the filters used by X-11.

In terms of seasonal adjustment, the X-11 style adjustment is characterised by smooth seasonal factors. Regardless of what happens in the original data, only cycles around the seasonal harmonics are included in the seasonal component.

### 3 Discussion of X11-Style Seasonal Adjustment

#### 3.1 Approximating X-11

Over the years X-11 has been extensively analysed (e.g. Young 1968, Cleveland 1972, Wallis 1982, Sutcliffe 1993, Dagum 1996), but it has proven difficult to find an exact representation of the program.

Young and Wallis left out steps in the actual X-11 process, such as the level correction step. The full iteration was accurately shown in Dagum. It is interesting to note that the weights underlying all of the X-11 filters, for any of the run options including ARIMA model extrapolations, can be obtained from the X-11 package by feeding it a series of “blip” time series and setting the run options to additive-with-no-outliers. Sutcliffe showed in 1993 how the outlier system used in X-11 could be incorporated into the linear approximation of X-11. The whole issue of how well X-11 can be approximated by a linear process is still unclear. The recent paper "Is Seasonal Adjustment a Linear or Nonlinear Data-Filtering Process?" by Ghysels et al (1995) investigated this issue.

### 3.2 Improvements to some of the X-11 basic filters

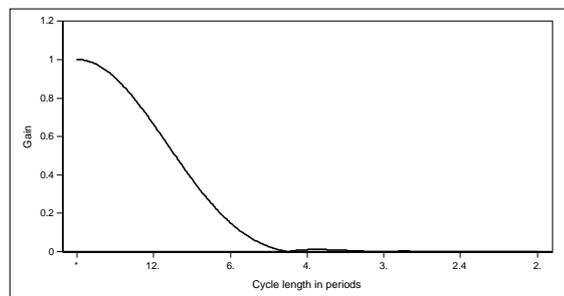
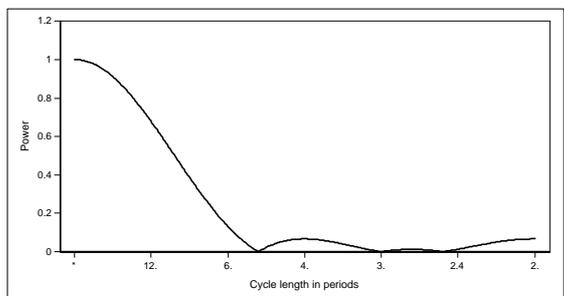
While X-11 is not perfect, it is a reasonable implementation of the basic philosophy of removing a frequency band around the seasonal harmonics. Slight improvements can bring it into line with equivalent model-based techniques.

The properties of symmetric filters can be better understood by plotting the gain function (which measures the effect of a linear filter on cycles of any period). A gain of 1 means that the amplitude of that cycle is unchanged, a gain of zero means the cycle is completely eliminated. As an example, GRAPH 3a shows the gain of the standard 3x5 filter used in X-11 and a modified 3x5 filter 3b. GRAPH 4a shows the convoluted gain of the final iteration symmetric linear filter of a standard X-11 run (13 term Henderson and 3x5 seasonal filter) and a slightly modified 3x5 seasonal filter that could be used in the X-11 program 4b.

GRAPH 3 Gain of Seasonal Filter used X-11

3a: Standard 3x5 (7 term)

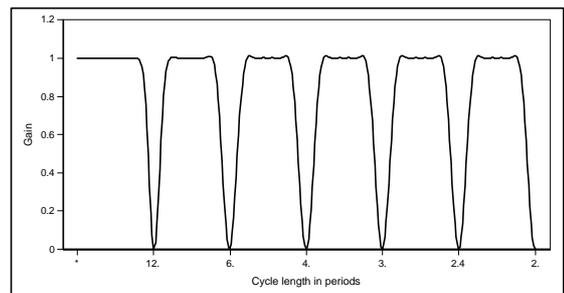
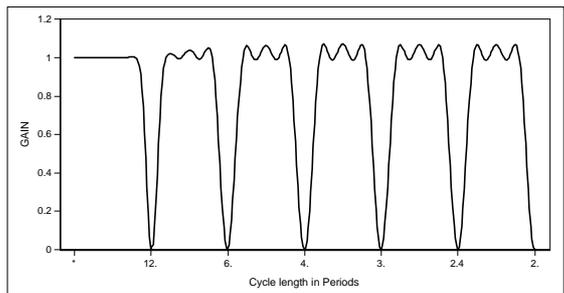
3b: Modified 3x5 (9 term)



GRAPH 4 Gain of X-11 Symmetric Adjustment Filter

4a: Standard X-11 (13H,3X5 Seasonal)

4b: Modified X-11 (13H, Mod Seasonal)



The slight modification shown in GRAPH 4b brings the gain of the X-11 seasonal adjustment filter very closely into line with model based versions with no amplification of cycles. It is also possible to use a model of the data to provide "exact" bands around the seasonal harmonics in closer accordance with the definition of the seasonal component.

A problem of X-11, that is sometimes highlighted by researchers, is that large outliers can dramatically affect the quality of the adjustment. Although X-11 has extensive and fairly good techniques for discounting outliers, outliers still have some impact on the adjustment process because X-11 uses an iterative procedure. There are three ways around this problem -- include more iterations in X-11 when required (the ABS uses this approach); use a regression approach such as used in X12 ARIMA; or use a model representation of X-11 to provide an "exact" discount for large outliers.

### 3.3 Multiplicative and pseudo-additive adjustments

The iterative nature of X-11 style methods enables a direct computation of a multiplicative model, defined by

$$3.3.1 \quad O_t = T_t * S_t * I_t.$$

For those who favour model-based adjustments, the issue of applying a true multiplicative model *versus* logging the data and applying an additive model-based seasonal adjustment requires further investigation. Generally the model-based solution is to log the data. This however gives geometric filters rather than linear filters which have a level bias. It is possible to provide true multiplicative versions of the model-based filters but only with iterative methods like X-11. Other solutions include correcting the final estimates for bias caused by the geometric weights of the log additive approach.

Other models, such as the pseudo-additive model of the form

$$3.3.2 \quad O_t = T_t * (S_t + I_t - 1)$$

are easily handled with an iterative technique, but very tricky in a model-based framework. The ABS has some national accounts data that is adequately adjusted using a pseudo additive model and X-11, but which gives very "bizarre" results using the automatic options in TRAMO/SEATS.

### 3.4 Practical use of X-11

X-11 is often portrayed as having fixed filters, yet modern versions can include a large range of trend and seasonal filters with end estimates tailored to the data. There is some logic to the X-11 approach of designing filters and forecasting methods from large amounts of actual data for application to data of very short duration. It may be preferable to use "global" weights derived from kindred long-span data series rather than attempt to estimate parameters and components from the small amount of data available for the actual series to be adjusted.

The extensive diagnostics provided by X-11 guide the user to a suitable adjustment; the standard version of the X-11 package requires a good deal of expert intervention. To make this process more "automatic", the Australian Bureau of Statistics has created a knowledge based seasonal adjustment package called SEASABS. This package attempts to provide options that reflect the "knowledge" gained in the ABS's time series analysis team over a 30 year period.

The practice (used in some journal articles) of feeding data through X-11 with options that clearly fall well outside the "guidelines" that a skilled analyst would apply to demonstrate how bad X-11 is, should not carry much evidentiary weight when doing a comparative assessment of adjustment. The US Bureau of the Census WWW site has the sensible injunction "This package is for use by knowledgeable users" in regard to its X12 ARIMA program.

X-11 has features not found in standard model based packages such as the ability to allow individual months' seasonal factors to evolve at different rates. The ABS has incorporated a moving trading day as standard because of continuing problems with a constant trading day regression.

Any statistical agency wanting to provide a quality seasonal adjustment service to its users has two major requirements:

- At least once a year three charts should be examined for each series to assess outliers behaviour, residual seasonality and residual trading day . These are a time series plot, the seasonal & irregular factors plotted by each month (these are affectionately known as worm charts at the ABS) and the trading day & irregulars plotted by type of month. These charts are demonstrated in appendix A as GRAPHS 8a, 8b and 8c. The x-axis is time for 8a, months for 8b and trading day type (eg all 31 day months ending in Monday, Tuesday and Wednesday) for 8c. The seasonal & irregular chart (8b) should be closely inspected for evidence of "residual" seasonality in particular months.
- Be aware that effects outside the scope of normal effects allowed for in time series packages may need attention. Some of those regularly handled at the ABS include ( amongst many others) abrupt changes in the seasonal pattern, moving trading day, and fortnightly payday effects.

## **4 Discussion of model based seasonal adjustment**

### **4.1 Basic seasonal component**

Two popular model based forms for the seasonal component are given below. Model 1 shows a form popular in many early model papers and model 2 a form made popular by the TRAMO/SEATS package. Model 1 has the “drawback” that as the innovation variance for the seasonal component gets large, the seasonal component includes power between the seasonal harmonics. This can be seen in GRAPH 5a. Model 2 forces at least the cycles between the seasonal harmonic to be not part of the seasonal regardless of the proportion of the total variance that the seasonal is. This is shown in GRAPH 5b. In model 2 the moving average parameters are computed to ensure a zero value midpoint between the seasonal harmonics, regardless of the seasonal hyper parameter.

### Model 1

Seasonal

$$4.1.1 \quad (1 + B + B^2 + B^3 \dots \dots \dots B^{11})S_t = ES_t \quad ES_t \text{ iid with variance } \sigma_s^2$$

Irregular

$$4.1.2 \quad I_t = EI_t \quad EI_t \text{ iid with variance } \sigma_i^2$$

### Model 2

Seasonal

$$4.1.3 \quad (1 + B + B^2 + B^3 \dots \dots \dots B^{11})S_t = (1 + 0.4B \dots 0.5B^{11})ES_t \quad \sigma_s^2$$

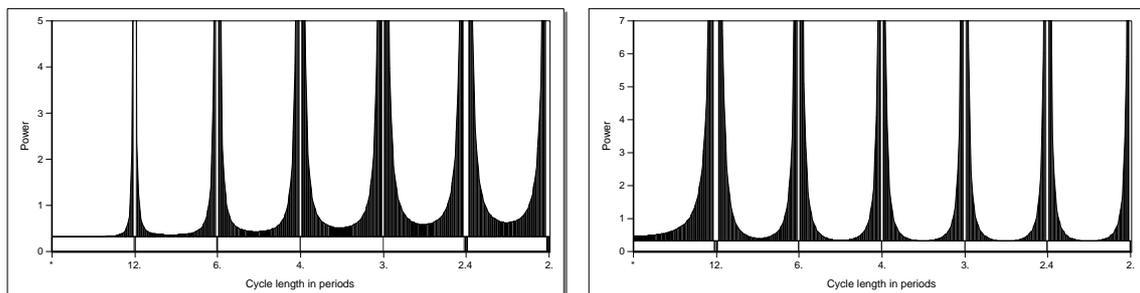
Irregular

$$4.1.4 \quad I_t = EI_t \quad \sigma_i^2$$

GRAPH 5 STOCHASTIC SEASONAL

5a. Model 1

5b. Model 2



## 4.2 Three component model

The basic “airline” type of model based seasonal adjustment can have two forms, namely as three separate models for trend, seasonal and irregular components, or as a composite model for the original time series. These are specified as follows:

## Form 1: Component models, total model derived from component models

Original

$$4.2.1 \quad O_t = T_t + S_t + I_t$$

Trend

$$4.2.2 \quad (1 - B)^2 T_t = ET_t \quad \sigma_T^2$$

Seasonal

$$4.2.3 \quad (1 + B + B^2 + \dots B^{11})S_t = ES_t \quad \sigma_S^2$$

Irregular

$$4.2.4 \quad I_t = EI_t \quad \sigma_I^2$$

Given a set of innovation variances, the component model given by 4.2.1-4.2.4 is equivalent to a composite model for the original data

$$4.2.5 \quad (1 - B)(1 - B^{12})O_t = (1 + B + \dots B^{11})ET_t + (1 - B)^2 ES_t + (1 - B)(1 - B^{12})EI_t$$

This model is equivalent in the sense that the non-stationary characteristics and autocorrelations are identical to the model given below (4.2.6) for a particular set of hyper parameters. For example if  $\sigma_T^2 = 1, \sigma_S^2 = 0.1, \sigma_I^2 = 1$  then the model is

Original

$$4.2.6 \quad (1 - B)(1 - B^{12})O_t = (1 - 0.5117B + 0.0416B^2 \dots - 0.6845B^{12} + 0.2751B^{13})EO_t$$

This is close to the standard airline model in Box Jenkins of

$$4.2.7 \quad (1 - B)(1 - B^{12})O_t = (1 - 0.5117B)(1 - 0.6845B^{12})EO_t$$

The estimation of the hyper parameters and components is usually done by using the Kalman filter.

## Form 2: Components computed from total model

Alternatively, starting with the airline model of the original data given in 4.2.7 a package such as SEATS would decompose the components as

Trend

$$4.2.8 \quad (1 - B)^2 T_t = (1 + 0.0311B - 0.9689B^2) E T_t$$

Seasonal

$$4.2.9 \quad (1 + B + B^2 + \dots B^{11}) S_t = (1 + 1.2539B + 1.2155B^2 - 0.3185B^{10} - 0.5527B^{11}) E S_t$$

Irregular

$$4.2.10 \quad I_t = E I_t$$

Typically this decomposition (4.2.8-4.2.10) looks like the model based solution shown in GRAPH 1a. Generally this alternative would be preferred by an official statistical agency such as the ABS since users are familiar with the seasonal component only including cycles around the seasonal harmonics. The alternative model (4.2.8-4.2.10) at least ensures that the power mid point between the seasonal harmonics in the seasonal component is zero regardless of the hyper parameter values.

### 4.3 Unusual definitions of the seasonal component

Packages such as TRAMO/SEATS have the ability to estimate a seasonal component that an X-11 style package would regard as being not in the spirit of the term seasonal adjustment as it is commonly used. The component model below gives one such decomposition, taken from the TRAMO/SEATS guidelines document.

Trend

$$4.3.1 \quad (1 - B) T_t = (1 + B) E T_t$$

Seasonal

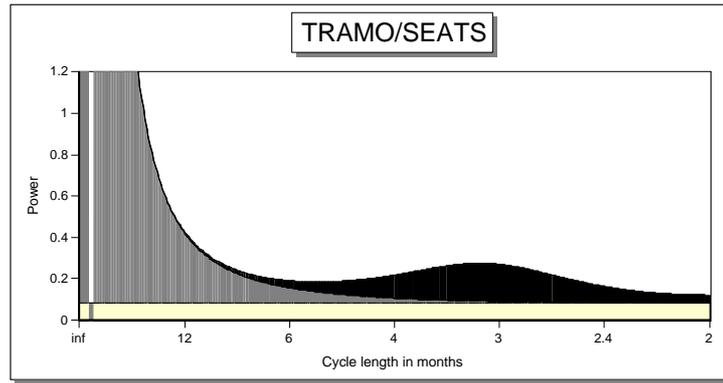
$$4.3.2 \quad (1 + 0.494B + 0.291B^2) S_t = (1 - 0.473B - 0.567B^2) E S_t$$

Irregular

$$4.3.3 \quad I_t = E I_t$$

GRAPH 6 shows graphically the decomposition of the total spectrum into the components. As before, white is the irregular, grey the trend and black the seasonal component. It can be noticed that while the seasonal component has a “peak” near the 3 month harmonic, it spreads over a large range of the spectrum. Such a seasonal component would be impossible with X-11.

GRAPH 6. An unusual decomposition.



#### 4.4 Spectral properties of model based methods

Such models as given in 4.2 are often referred as the Basic Structural Model (BSM). The power spectrum of the BSM component estimates as given in 4.2.2-4.2.4 can be readily computed as

Trend

$$4.4.1 \quad S_T(w) = (\sigma_T^2/\pi) * ((1 - 2 \cos(w) + \cos(2w))^2 + (-2 \sin(w) + \sin(2w))^2)^{-1}$$

Seasonal

$$4.4.2 \quad S_S(w) = (\sigma_S^2/\pi) * ((\sum_{j=0}^{11} \cos(jw))^2 + (\sum_{j=0}^{11} \sin(jw))^2)^{-1}$$

Irregular

$$4.4.3 \quad S_I(w) = (\sigma_I^2/\pi)$$

If the power spectrum of the original data given by 4.2.6 is computed we have

$$4.4.4 \quad S_O(w) = S_T(w) + S_S(w) + S_I(w)$$

Generally when estimating the components a MMSE criterion for the linear filters is used. That is, linear weights that when applied to the original data minimise the expected mean square error between the filtered original and the relevant component given the model. It can be shown that the implied gain of the theoretical symmetric linear filters (the end filters are given by the model forecasts) given by this criteria are

Trend

$$4.4.5 \quad G_T(w) = S_T(w)/S_O(w)$$

Seasonal

$$4.4.6 \quad G_S(w) = S_S(w)/S_O(w)$$

Irregular

$$4.4.7 \quad G_I(w) = S_I(w)/S_O(w)$$

It can be seen from 4.4.5-4.4.7 that because all  $S(w)$  are non-negative the model based gains must be between 0 and 1. Also the MMSE criterion when applied to obtain estimates will not exactly reproduce the component models. This is because the output spectrum of a linear filter is the gain squared by the input spectrum (see Jenkins, 1968). For example applying the MMSE linear filter to the original data to obtain the trend gives an output spectrum of

$$4.4.8 \quad G_T(w) * G_T(w) * S_O(w) = [S_T(w)/S_O(w)]^2 S_O(w) = S_T(w) * [S_T(w)/S_O(w)]$$

A similar result is obtained for the other components. Hence the estimate of the trend component spectrum will differ from the actual component spectrum by the factor  $S_T(w)/S_O(w)$ .

The effect of this is shown in GRAPH 2a for the residual. To obtain the actual component spectrums one would need to find linear filters that gave the square roots of the gains given in 4.4.5-4.4.7. Unfortunately if this was attempted the components would no longer add up to the original data.

#### 4.5 An alternative to the MMSE criterion

It is interesting to note that one could, for example, have a white noise irregular by computing the irregular linear filter to have a gain of the form

Irregular

$$4.5.1 \quad G_I(w) = \sqrt{S_I(w)/S_O(w)}$$

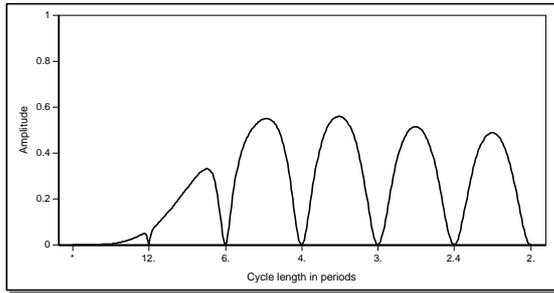
and the trend and seasonal such that

$$4.5.2 \quad G_T(w) + G_S(w) = 1 - G_I(w).$$

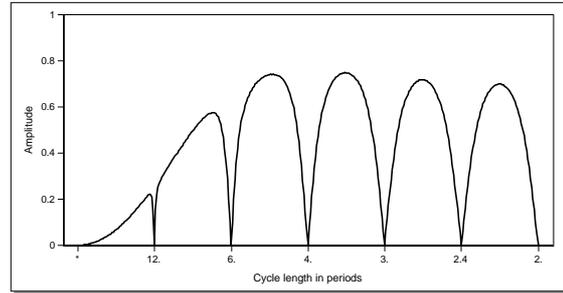
Such a filter could be called the SQRT criteria, however the MMSE criteria would be sacrificed. Graph 7 shows the periodogram for the residual component computed using the standard MMSE and SQRT criteria, using the BSM to simulate data. In this case the hyper parameters selected are  $\sigma_T^2 = 1, \sigma_S^2 = 1, \sigma_I^2 = 1$

## GRAPH 7 BSM

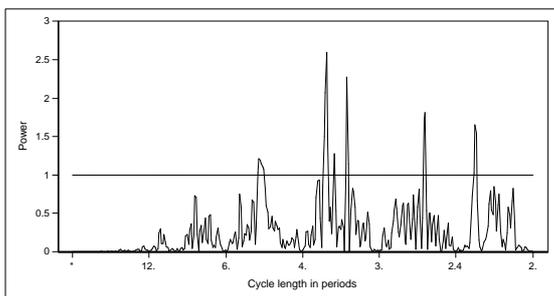
7a. Gain MMSE criteria



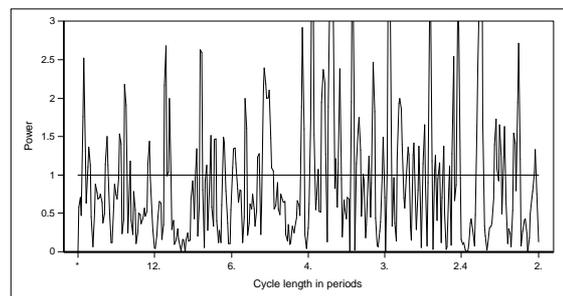
7b. Gain Sqrt criteria



7c. Periodogram of MMSE irregular



7d. Periodogram of Sqrt irregular



As can be seen, the MMSE gives an irregular with the long cycles damped significantly. There is also noticeable damping at the seasonal cycles. In contrast, the modified criterion gives a much more white noise looking periodogram.

The degree of distortion of the irregular component depends on the hyper parameters. Generally the more deterministic the trend and seasonal the less the distortion. A disadvantage of the Sqrt linear filter is that it is complicated to compute in contrast to the MMSE filter, where several neat methods exist. However, if a component that has the spectral characteristics of the component model is required, then this technique is useful.

## 5 Final comments

When a statistical agency decides on a seasonal adjustment technique, the key users of the results (such as economic policy departments and influential commentators) are the most important. In the ABS's experience, their needs are met well by X-11 style adjustments.

Modellers have special concerns. There are clear cases where a model based method is required -- for example, when building an econometric model that abstracts from seasonal influences. Some researchers would feel uneasy about feeding into their models seasonally adjusted data derived from a package such as X-11 (no matter how well the parameters have been selected) because the seasonally adjusted data at any point in time exploits information contained in future values of the data. These users may prefer to incorporate a model of the seasonal process within their econometric models.

For some applications, a user may wish to incorporate a known or assumed model of a component into the seasonal adjustment process. For example, recent research work at the ABS has specified and incorporated models of the sampling error. X-11 is less amenable to this sort of application.

For general users of seasonally adjusted data who are expecting a smooth trend and seasonal component regardless of the characteristics of the data, an X-11 style adjustment is recommended. Such users are also not expecting the seasonal component to contain any cycles spread between the seasonal. That said, it is, however, important that the best possible procedure is used to obtain estimates at the ends of the data. Providing this is met by an X-11 style adjustment (eg using X12 ARIMA), the seasonal adjustment is perfectly valid. Given the model shown in GRAPH 1 and many other models the decomposition into trend, seasonal and residual is equally valid given different criteria.

For some data, the model based approach may give seasonal components dramatically different to that provided by X-11 as outlined in section 4.3.

Both the model based and X-11 style “camps” can learn from each other. It may be 10-20 years before we see a standard, agreed and integrated approach to time series decomposition. Even then human experts will still be required to sort out difficult cases.

## 6. References

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APPENDIX A

